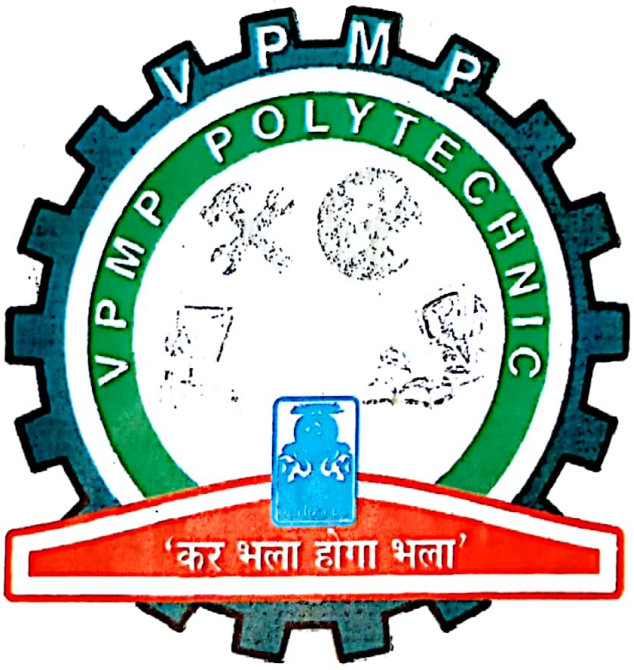
VPMP POLYTECHNIC, GANDHINAGAR

BASIC MATHEMATICS

CODE NO: - 3300001



QUESTION BANK

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UNIT-I

MARKS-10

LOGARITHM

(MCQS-03MARKS,EXAMPLES-07 MARKS)

* MCQ'S.

1) $\log_{15} \frac{1}{}$ = ____

2) $\log_2 8$ = ____

3) $\log_b^a \times \log_a^b$ = ____

4) $\log 1 \cdot \log 2 \cdot \log 3 \cdot \log 4 \cdot \log 5$ = ____

5) If $\log_7 x = 1$ then x = ____

6) $\log(\tan\theta) + \log(\cot\theta)$ = ____

7) If $\log(\frac{a}{b}) + \log(\frac{b}{a}) = \log(a+b)$ then ____

8) value of $\log_a(\frac{1}{a})$ = ____

9) $\log 81 \div \log 27$ = ____

10) $\log 32 \div \log 16$ = ____

11) $a^{\log_a b}$ = ____

12) $\log_5 125$ = ____

13) If $\log_a 32 = 5$ then a = ____

14) $\log_{10} 0.0001$ = ____

15) $2^{-\log_2 3}$ = ____

16) If $\log_2 x = 5$ then x = ____

17) If $\log x + \log 2x = \log 18$ then x = ____

18) $1024^{\log_2 m}$ = ____

19) If $\log_3(\log_2 x) = 1$ then x = ____

20) $\log(a) + \log(\frac{1}{a})$ = ____

21) If $\log_{10}(x+1) + \log_{10}(x-1) = \log_{10} 3$ then x = ____

22) $\log 27 \div \log 9$ = ____

23) If $a^x = b^y$ then $\frac{x}{y}$ = ____

24) If $\log_x(\frac{9}{16}) = -2$ then x = ____

25) $\log_4(\frac{1}{2})$ = ____

26) $\log m - \log n$ = ____

27) $\log_5 625 + \log_5 \sqrt{5}$ = ____.

* Examples :-

1) Simplify :

$$\log 2 + 16 \log\left(\frac{16}{15}\right) + 12 \log\left(\frac{25}{24}\right) + 7 \log\left(\frac{81}{80}\right)$$

2) Prove that :-

$$2 \log\left(\frac{5}{7}\right) + \frac{1}{2} \log\left(\frac{81}{16}\right) - \log\left(\frac{27}{196}\right) = \log 12.$$

3) Prove that :-

$$\log_{10} 800 = 2 + 3 \log_{10} 2 .$$

4) Prove that :-

$$\frac{1}{\log_2 6} + \frac{1}{\log_3 6} = 1$$

5) Prove that :-

$$\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2$$

6) Prove that :-

$$\log_a P + \log_{a^2} P^2 + \log_{a^3} P^3 + \log_{a^4} P^4 = 4 \log_a P$$

7) Prove that :-

$$\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} = 2$$

8) Prove that :-

$$\frac{1}{\log_x yz + 1} + \frac{1}{\log_y zx + 1} + \frac{1}{\log_z xy + 1} = 1$$

9) Prove that :-

$$\log\left[x + \sqrt{x^2 - 1}\right] + \log\left[x - \sqrt{x^2 - 1}\right] = 0$$

10) If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then prove that $a = b$

11) If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then prove that $a^2 + b^2 = 2ab$.

12) If $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$ then prove that $x^2 + y^2 = 7xy$ or $\frac{x}{y} + \frac{y}{x} = 7$.

13) Solve :-

$$\log_2(\log_3(\log_2 x)) = 1$$

14) If $\frac{\log x \times \log 16}{\log 32} = \log 256$ then find the value of x.

15) If $\frac{4 \log 3 \times \log x}{\log 9} = \log 27$ then find the value of x.

16) Solve :-

$$\log_2(x+5) + \log_2(x-2) = 3$$

17) Solve :-

$$\log(x) + \log(x-5) = \log 6 .$$

18) If $a^x = b^y = c^z$ then Prove that $\log_a bc = x\left(\frac{1}{y} + \frac{1}{z}\right)$, where $x, y, z \neq 0$.

19) Prove that

$$\log\left[\sqrt{x^2+1} + x\right] + \log\left[\sqrt{x^2+1} - x\right] = 0$$

20) If $\log(x+y) = \log 3 + \frac{1}{2}\log x + \frac{1}{2}\log y$ then P.T. $x^2 + y^2 = 7xy$

21) Prove that

$$\log_b a \cdot \log_c b \cdot \log_a c = 1$$

UNIT-II

MARKS-18

DETERMINAT

&

MATRICES

(MCQS-04MARKS,EXAMPLES-14MARKS)

*MCQ'S.

1) If $A=\begin{bmatrix}\sec\theta & \tan\theta\\ \tan\theta & \sec\theta\end{bmatrix}$ then $|A|=$ ____

2) $\begin{vmatrix}1 & \log_y x\\ \log_x y & 1\end{vmatrix}=$ ____

3) The value of $\begin{vmatrix}\log_6 3 & -1\\ \log_6 2 & 1\end{vmatrix}=$ ____

4) $\begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix}+\begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}=$ ____

5) order of $\begin{bmatrix}2 & 1 & 2\\ 1 & 1 & 1\end{bmatrix}$ is ____

6) If $\begin{vmatrix}x & 1\\ 4 & 2\end{vmatrix}=0$ then $x=$ ____

7) If $\begin{vmatrix}x & 3\\ -2 & 2\end{vmatrix}=2$ then $x=$ ____

8) If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ then $A^T=$ ____

9) If $A=\begin{bmatrix}1 & -3 & 4\\ -2 & 1 & 2\end{bmatrix}$ then $A^T=$ ____

10) If $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ then $A^T=$ ____

11) If $A=\begin{bmatrix}1 & 2\\ 3 & 1\\ 4 & 2\end{bmatrix}$ then $A^T=$ ____

12) If $A=\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix}$ then $AB=$ ____

13) If $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ then $A^2=$ ____

14) If $A=\begin{bmatrix}-7 & 6\\ 5 & -2\end{bmatrix}$ then $AI=$ ____

15) If $A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}$ then cofector of $5=$ ____

16) If $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ then adj $A=$ ____

17) If $A=\begin{bmatrix}-8 & 4\\ -6 & 3\end{bmatrix}$ then $A^{-1}=$ ____

18) If $\begin{bmatrix}3 & 2\\ x-1 & 5\end{bmatrix}=\begin{bmatrix}3 & y+1\\ 4 & 5\end{bmatrix}$ then $(x,y)=$

19) If $\begin{vmatrix}a & b\\ c & d\end{vmatrix}=5$ then $\begin{vmatrix}3a & 3b\\ 3c & 3d\end{vmatrix}=$ ____

20) $\begin{vmatrix}\log_e e & \log_{10} 10\\ 4 & 4\end{vmatrix}=$ ____.

21) $\begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix}=$ ____.

22) If $A=\begin{bmatrix}1 & 4\\ 3 & -2\end{bmatrix}$ then $2A-3I=$ ____

23) If $\begin{bmatrix}0 & x & -2\end{bmatrix}\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}=[4]$ then $x=$ ____.

24) $\begin{vmatrix}2 & -3\\ 5 & 4\end{vmatrix}=$ ____.

25) If $A=\begin{bmatrix}1 & -2\\ 2 & -1\end{bmatrix}$ then Adj $A=$ ____.

26) If $A = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$ then $3A =$ ____.

27) If $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

then $AB =$ ____.

*Examples:-

1) If $\begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$ then find x.

2) If $A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 3 & 5 \\ -1 & 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 7 \\ 2 & 3 & -4 \end{bmatrix}$

then find $2A + 3B$.

3) If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$

then find $3A - 2B$.

4) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then prove that

$A^2 - (a+d)A + (ad - bc)I = 0$.

5) If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ then show

that $A^2 = A$.

6) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix}$

Find AB or BA whichever exist.

7) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$

then find AB and BA.

8) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then prove that

$A^2 - 5A - 2I = 0$.

9) If $A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ then

find $A^2 - 2A - I$.

10) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then prove tha

$A^2 - 4A + 7I = 0$.

11) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show

that $A^2 - 4A - 5I = 0$.

12) If $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 2 & 1 \end{bmatrix}$

then find $(AB)^T$.

13) If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$

then prove that $(A+B)^T = A^T + B^T$

14) If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then

Prove that $\text{adj}\, A = A$.

15) If $A=\begin{bmatrix}5 & -3\\ 2 & 1\end{bmatrix}$ then find A^{-1}.

16) If $A=\begin{bmatrix}3 & 5\\ -2 & -3\end{bmatrix}$ then find A^{-1}.

17) If $A+B=\begin{bmatrix}1 & -1\\ 3 & 0\end{bmatrix}$, $A-B=\begin{bmatrix}3 & 1\\ 1 & 4\end{bmatrix}$ then find $(AB)^{-1}$.

18) Find A^{-1} if exists for

$$A=\begin{bmatrix}1 & 2 & 1\\ 3 & 2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

19) Find inverse of matrix $\begin{bmatrix}3 & -1 & 2\\ 4 & 1 & -1\\ 5 & 0 & 1\end{bmatrix}$

20) If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 1\end{bmatrix}$ then find A^{-1}.

21) If $A=\begin{bmatrix}3 & -10 & -1\\ -2 & 8 & 2\\ 2 & -4 & -2\end{bmatrix}$ then find A^{-1}.

22) Solve the equation $3x-y=1$, and $x+2y=5$ using matrix method.

23) Solve the equation $2x+5y=7$, and $8x-3y=5$ using matrix method.

24) Solve the equation $2x+3y=1$, and $y-4x=2$ using matrix method.

25) Solve the equation $2x-3y=-5$ and $3x+y=9$ using matrix method.

26) Solve the equation $3x+2y=7$ and $11x-4y=3$ using matrix method.

27) From Equation $\begin{bmatrix}x & 3\\ y & 2\end{bmatrix}\begin{bmatrix}2\\ 3\end{bmatrix}=\begin{bmatrix}15\\ 12\end{bmatrix}$ Find value of x and y.

28) If $A=\begin{bmatrix}1 & 2 & 1\\ 3 & 4 & 2\end{bmatrix}$, $B=\begin{bmatrix}3 & -2 & 4\\ 1 & 5 & 0\end{bmatrix}$

Find matrix X from $X+A+B=0$

29) If $A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ then Prove that $A^4=I$ using matrix.

30) solve equations $3x+2y=5$ and $2x-y=1$ using matrix.

31) Expand $\begin{vmatrix}5 & 3 & -1\\ 4 & -3 & 0\\ 6 & 1 & 2\end{vmatrix}$ using Sarrus's method

32) If $A=\begin{bmatrix}2 & -2\\ 3 & 1\end{bmatrix}$, $B=\begin{bmatrix}-1 & 5\\ 4 & -3\end{bmatrix}$

then P.T. $(AB)^T=B^TA^T$

33) If $A=\begin{bmatrix}3 & 1 & 2\\ 2 & -3 & -1\\ 1 & 2 & 1\end{bmatrix}$ then find A^{-1}.

UNIT-III

MARKS-18

TRIGONOMERY

(MCQS-04MARKS,EXAMPLES-14 MARKS)

* MCQ'S.

1) $\frac{4\pi}{9}$ radian = ____ degree.

2) 540° = ____ radian.

3) 135° = ____ radian.

4) Period of $\cos(2x+7)$ is ____

5) Principal Period of $\cos\left(\frac{2x}{3}+5\right)$ = ____

6) Period of $\sin(2x+3)$ = ____.

7) The Period of $\tan 3x$ = ____.

8) The Period of $3\cos 2x$ is ____.

9) $\cos\frac{\pi}{6}\cdot\cos\frac{\pi}{4}\cdot\cos\frac{\pi}{3}\cdot\cos\frac{\pi}{2}$ = ____.

10) $\cos\frac{\pi}{2}\cdot\sin\frac{3\pi}{2}\cdot\sin\frac{5\pi}{2}$ = ____.

11) $\sin^2 42^\circ + \sin^2 48^\circ$ = ____

12) $\sin^2 35^\circ + \sin^2 55^\circ$ = ____.

13) $\sin^2 40^\circ + \sin^2 50^\circ$ = ____.

14) $\sin\frac{\pi}{8} + \sin\frac{4\pi}{8}$ = ____.

15) $\sin 40^\circ + \sin 20^\circ$ = ____.

16) $\sin\frac{\pi}{8} + \sin\frac{9\pi}{8}$ = ____.

17) $\cos(\pi+\theta)$ = ____.

18) $\tan(\pi+\theta)$ = ____.

19) $\sin 120^\circ$ = ____.

20) $\sin(A+B)\cdot\sin(A-B)$ = ____

21) $\sin 3A$ = ____.

22) $\cos 3A$ = ____.

23) If $\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$ then $\sin 165^\circ$ = ____

24) $\sin^{-1}(\cos\frac{\pi}{3})$ = ____.

25) $\tan^{-1}x + \cot^{-1}x$ = ____

26) $\tan^{-1}(\sqrt{3})$ = ____.

27) $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{4}{3}\right)$ = ____.

28) value of $\cos\left(2\tan^{-1}\left(\frac{1}{2}\right)\right)$ = ____.

29) $\sin\frac{\pi}{8}$ = ____.

30) period of $\cot\frac{x}{6}$ = ____.

31) If $\tan\theta = \frac{3}{4}$ then $\tan 2\theta$ = ____.

32) $\sin 135^\circ$ = ____.

33) $\sin^{-1}x + \cos^{-1}x$ = ____.

34) period of $\sin 3x$ = ____.

* Examples :-

1) Simplify :

$$\frac{\sin\left(\frac{\pi}{2}+\theta\right)}{\cos(\pi-\theta)} + \frac{\cot\left(\frac{3\pi}{2}-\theta\right)}{\tan(\pi-\theta)} + \frac{\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)}{\sec(\pi+\theta)}$$

2) Evaluate :

$$\frac{\sin\left(\theta-\frac{\pi}{2}\right)}{\cos\left(\theta-\frac{\pi}{2}\right)} + \frac{\tan\left(\frac{\pi}{2}+\theta\right)}{\cot(\pi+\theta)} + \frac{\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)}{\sec(\pi+\theta)}$$

3) Find the value of

$$\frac{\sin\left(\theta-\frac{\pi}{2}\right)}{\cos(\theta-\pi)} + \frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos(\pi-\theta)} + \frac{\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)}{\sec(\pi+\theta)}$$

4) Prove that :

$$\frac{\sin(-\theta)\cdot\tan\left(\frac{\pi}{2}+\theta\right)\cdot\sin(\pi+\theta)\cdot\sec\left(\frac{3\pi}{2}+\theta\right)}{\sin(2\pi-\theta)\cdot\cos\left(\frac{3\pi}{2}-\theta\right)\cdot\operatorname{cosec}(\pi-\theta)\cdot\cot(2\pi-\theta)}$$

5) Simplify :

$$\frac{\sin(180^\circ-\theta)\cdot\cos(270^\circ-\theta)\cdot\operatorname{cosec}(90^\circ+\theta)}{\sec(270^\circ+\theta)\cdot\cot(90^\circ+\theta)\cdot\tan(360^\circ+\theta)}$$

6) Evaluate :

$$\frac{\sin\left(\theta-\frac{\pi}{2}\right)}{\cos(\theta-\pi)} + \frac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cot(2\pi+\theta)} + \frac{\operatorname{cosec}\left(\frac{3\pi}{2}-\theta\right)}{\sec(\pi-\theta)}$$

7) Prove that :

$$\cos\frac{19\pi}{6}\cdot\sin\frac{17\pi}{6} - \sin\frac{11\pi}{6}\cdot\cos\frac{13\pi}{6} = 0$$

8) Prove that :

$$\sin^2\frac{\pi}{4} + \sin^2\frac{3\pi}{4} + \sin^2\frac{5\pi}{4} + \sin^2\frac{7\pi}{4} = 2$$

9) Prove that :

$$\tan\left(\frac{5\pi}{4}\right)\cdot\cot\left(\frac{9\pi}{4}\right)\cdot\tan\left(\frac{17\pi}{4}\right)\cdot\cot\left(\frac{15\pi}{4}\right) = -1$$

10) Prove that :

$$\tan\frac{\pi}{20}\cdot\tan\frac{3\pi}{20}\cdot\tan\frac{5\pi}{20}\cdot\tan\frac{7\pi}{20}\cdot\tan\frac{9\pi}{20} = 1$$

11) Prove that :

$$\cot\frac{\pi}{20}\cdot\cot\frac{3\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\cot\frac{7\pi}{20}\cdot\cot\frac{9\pi}{20} = 1.$$

12) Draw the graph of

$$y = \sin x, \quad 0 \le x \le \pi$$

13) Draw the graph of

$$y = \sin x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}.$$

14) Draw the graph of

$$y = \sin 2x, \quad 0 \le x \le \pi.$$

15) Draw the graph of

$$y = \sin\frac{x}{2}, \quad 0 \le x \le 2\pi.$$

16) Draw the graph of

$$y = \cos x, \quad 0 \le x \le \pi$$

17) Draw the graph of

$$y = \cos x, \quad 0 \le x \le 2\pi$$

18) Draw the graph of

$$y = \cos\frac{x}{2}, \quad 0 \le x \le 2\pi$$

19) Prove that :-

$$\cos A\cdot\sin(B-C) + \cos B\cdot\sin(C-A) + \cos C\cdot\sin(A-B) = 0.$$

20) Prove that :-

$$\tan 57^\circ = \frac{\cos 12^\circ + \sin 12^\circ}{\cos 12^\circ - \sin 12^\circ}$$

21) Prove that :-

$$\tan 66^\circ = \frac{\cos 21^\circ + \sin 21^\circ}{\cos 21^\circ - \sin 21^\circ}$$

22) Prove that :

$$(1+\tan 25^\circ)(1+\tan 20^\circ) = 2$$

23) Prove that :-

$$\tan 20^\circ + \tan 25^\circ + \tan 20^\circ\cdot\tan 25^\circ = 1$$

24) Prove that :-

$$\tan 50^\circ = \tan 40^\circ + 2\tan 10^\circ$$

25) Prove that :-

$$\frac{\sin(A-B)}{\cos A\cdot\cos B} + \frac{\sin(B-C)}{\cos B\cdot\cos C} + \frac{\sin(C-A)}{\cos C\cdot\cos A} = 0$$

26) Prove that :-

$\tan\alpha + \tan\beta = \sec\alpha \cdot \sec\beta \cdot \sin(\alpha+\beta)$

27) For ΔABC Prove that

$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

28) Prove that :-

$\tan 70^\circ - \tan 50^\circ - \tan 20^\circ = \tan 70^\circ \cdot \tan 50^\circ \cdot \tan 20^\circ$

29) Prove that :-

$\tan 5A - \tan 3A - \tan 2A = \tan 5A \cdot \tan 3A \cdot \tan 2A$

30) Prove that :-

$$\frac{\sin\theta + \sin 2\theta + \sin 4\theta + \sin 5\theta}{\cos\theta + \cos 2\theta + \cos 4\theta + \cos 5\theta} = \tan 3\theta.$$

31) Prove that :-

$$\frac{\cos A + \cos 3A + \cos 5A}{\sin A + \sin 3A + \sin 5A} = \cot 3A.$$

32) Prove that :-

$$\frac{\cos 3A + 2\cos 5A + \cos 7A}{\sin 3A + 2\sin 5A + \sin 7A} = \cot 5A.$$

33) Prove that :-

$\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \frac{1}{16}$

34) Prove that :-

$8 \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ = 1$

35) Prove that :-

$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$

36) Prove that :-

i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$

ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$

iii) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$

37) If $\tan\theta = \frac{1}{2}$ then Prove that

$7\cos 2\theta + 8\sin 2\theta = \frac{53}{5}$

38) If $\tan\theta = \frac{2}{3}$, then find the value of $2\sin 2\theta + 3\cos 2\theta$.

39) Prove that :-

$$\frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \tan A.$$

40) Prove that :-

$$\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\frac{\theta}{2}.$$

41) Prove that :-

$$\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$$

42) Prove that :-

$\sin 4\theta = 4\sin\theta \cdot \cos^3\theta - 4\sin^3\theta \cdot \cos\theta$

43) If $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$, then Prove that $\cos 2\theta = \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)$

44) Prove that :-

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $|x| \le 1$.

45) Find the value of

$\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right)$

46) Prove that :-

$\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{2}.$

47) Prove that :-

$\tan^{-1}(\infty) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{7\pi}{6}.$

48) Prove that :-

$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}.$

49) Prove that :-

$\cos^{-1}\left(\frac{2}{\sqrt{5}}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}.$

50) Prove that :-

$$2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\frac{\pi}{4}$$

51) Prove that :-

$$2\tan^{-1}\left(\frac{2}{3}\right)=\tan^{-1}\left(\frac{12}{5}\right)$$

52) Prove that :-

$$\tan^{-1}\left(\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{1}{7}\right)$$

53) Prove that :-

$$\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\tan^{-1}\left(\frac{1}{2}\right).$$

54) Find value of $\tan\left(2\tan^{-1}\frac{1}{3}\right)$.

55) Prove that

$$\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A}=\tan A.$$

56) prove that

$$\cos\frac{\pi}{8}+\cos\frac{3\pi}{8}+\cos\frac{5\pi}{8}+\cos\frac{7\pi}{8}=0.$$

57) If $a\sin\theta-b\cos\theta=0$ then prove that $a\cos 2\theta+b\sin 2\theta=a$

58) Find $\sin 15^\circ$

59) Prove that

$$\sin(A+B)\cdot\sin(A-B)=\sin^2A-\sin^2B.$$

60) prove that

$$\frac{\sin(\pi+\theta)}{\sin(2\pi-\theta)}+\frac{\tan\left(\frac{\pi}{2}+\theta\right)}{\cot(\pi-\theta)}+\frac{\cos(2\pi+\theta)}{\sin\left(\frac{\pi}{2}+\theta\right)}=3.$$

61) prove that

$$\frac{\sin 4A+2\sin 5A+\sin 6A}{\cos 4A+2\cos 5A+\cos 6A}=\tan 5A$$

62) Prove that $\sin(\tan^{-1}x+\cot^{-1}x)=1$.

UNIT-IV

MARKS-14

VECTOR

(MCQS-00MARKS,EXAMPLES-14 MARKS)

* Example :

1) If $\bar{a} = j + k - i$ and $\bar{b} = 2i + j - 3k$ then find $|2\bar{a} + 3\bar{b}|$

2) If $\bar{a} = (1,2,1)$, $\bar{b} = (1,-1,2)$, $\bar{c} = (3,2,-1)$ then find $|3\bar{a} + \bar{b} - 2\bar{c}|$

3) If $\bar{a} = (1,1,1)$, $\bar{b} = (2,1,2)$, $\bar{c} = (-1,0,3)$ then find $|2\bar{a} + \bar{b} - \bar{c}|$

4) If $\bar{a} = (3,-1,-4)$, $\bar{b} = (-2,4,-3)$, $\bar{c} = (-1,2,-5)$ then find magnitude of $\bar{a} + 2\bar{b} - \bar{c}$.

5) If $\bar{a} = 5i - 3j + 2k$, $\bar{b} = 2i + 3j - k$ and $\bar{c} = i + 2j + 3k$ then find $|2\bar{a} - 3\bar{b} + 4\bar{c}|$

6) If $\bar{a} = 3i - 2j + k$, $\bar{b} = 2i - 4j - 3k$ and $\bar{c} = -i + 2j + 2k$ then find $|2\bar{a} - 3\bar{b} - 5\bar{c}|$

7) If $\bar{a} = i + 2j - k$, $\bar{b} = 3i + j + 2k$ and $\bar{c} = -2i - j + 5k$ then find $|2\bar{a} + 3\bar{b} - \bar{c}|$

8) If $\bar{a} = (3,-1,-4)$, $\bar{b} = (-2,4,-3)$, $\bar{c} = (-1,2,1)$ then find $|3\bar{a} - 2\bar{b} + 4\bar{c}|$

9) If $\bar{a} = 3i - j - 4k$, $\bar{b} = -2i + 4j - 3k$ and $\bar{c} = i + 2j - k$ then find the direction cosines of the vector $3\bar{a} - 2\bar{b} + 4\bar{c}$.

10) If $\bar{a} = (-4,9,6)$, $\bar{b} = (0,7,10)$, $\bar{c} = (-1,6,6)$ then show that $(\bar{a} - \bar{c}) \cdot (\bar{b} - \bar{c}) = 0$

11) If $\bar{x} = (1,-2,3)$ and $\bar{y} = (-2,3,1)$ then find $(\bar{x} + \bar{y}) \cdot (\bar{x} - \bar{y})$.

12) If $\bar{x} = (1,-2,3)$ and $\bar{y} = (1,2,-2)$ then find $(\bar{x} + \bar{y}) \cdot (\bar{x} - \bar{y})$.

13) Find α, if $\bar{a} = (2,-3,5)$ and $\bar{b} = (x,-6,-8)$ are perpendicular to each other.

14) If $\bar{x} = (1,-2,-3)$ and $\bar{y} = (2,P,4)$ then For what value of P vectors $\bar{x}$ and $\bar{y}$ are perpendicular to eachother.

15) For what value of m, the vectors $2i - 3j + 5k$ and $mi - 6j - 8k$ are perpendicular to each other?

16) For what value of p, the vectors $2i + 3j + k$ and $pi - j - 3k$ are perpendicular to each other?

17) If $(m, 2m, 4)$ and $(m, -3, 2)$ are perpendicular to each other then find m.

18) simplify :

$(10i + 2j + 3k) \cdot [(i - 2j + 2k) \times (3i - 2j - 2k)]$

19) If $\bar{a} = 2i - j$ and $\bar{b} = i + 3j - 2k$, then find $|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})|$

20) If $\bar{a} = (2,-3,-1)$ and $\bar{b} = (1,4,-3)$ then find $(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})$. Also find modulus of $(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})$.

21) If $\bar{x} = 3i - j + 2k$ and $\bar{y} = 2i + j - k$ then find the unit vector perpendicular to both $\bar{x}$ and $\bar{y}$.

22) Find a unit vector perpendicular to the both vectors $\bar{a} = (5,7,-2)$ and $\bar{b} = (3,1,-2)$

23) Find the unit vector perpendicula the plane of vectors $\bar{a} = (1,2,3)$ and $\bar{b} = (-2,1,-2)$.

24) Find the unit vector perpendicular to $\bar{a} = (3,1,2)$ and $\bar{b} = (2,-2,4)$

25) Find the unit vector perpendicular to both $\bar{a} = (1,-1,1)$ and $\bar{b} = (2,3,-1)$

26) If $\bar{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\bar{b} = \hat{i} - \hat{j} + \hat{k}$, then find unit vector perpendicular to $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$.

27) If $\bar{x} = (1,1,1)$ and $\bar{y} = (2,-1,-1)$ then prove that $\bar{x}$ is perpendicular to $\bar{y}$. Also find unit vector perpendicular to both $\bar{x}$ and $\bar{y}$.

28) Find the angle between $(1,2,4)$ and $(3,1,2)$.

29) Find the angle between $(1,2,3)$ and $(-2,3,1)$.

30) Find the angle between two vectors $2\hat{i} + \hat{j} - 3\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$.

31) Show that the angle between two vectors $\hat{i} + \hat{j} - \hat{k}$ and $2\hat{i} - 2\hat{j} + \hat{k}$ is $\sin^{-1}\sqrt{\frac{26}{27}}$.

32) Prove that the angle between two vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$ is $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$.

33) Prove that the angle between two vectors $\hat{i} + 2\hat{j}$ and $\hat{i} + \hat{j} + 3\hat{k}$ is $\sin^{-1}\sqrt{\frac{46}{55}}$.

34) Prove that the angle between two vectors $\bar{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\bar{b} = 2\hat{i} + \hat{j} - \hat{k}$ is $\sin^{-1}\sqrt{\frac{35}{84}}$.

35) The forces $3\hat{i} - 2\hat{j} + \hat{k}$ and $-\hat{i} - \hat{j} + 2\hat{k}$ act on a particle and particle moves from the point $(2,2,-3)$ to the point $(-1,2,4)$ under the effect of these forces find work done.

36) A particle moves from $(-1,2,1)$ to $(2,3,-1)$ under the effect of the forces $(1,2,1)$ and $(2,-1,0)$ find work done.

37) The constant forces $(1,2,3)$ and $(3,1,1)$ act on a particle under the action of these forces particle move to the point $(5,1,2)$ from the point $(0,1,-2)$ Find the total work done.

38) The forces $3\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{k} + \hat{i} + 5\hat{j}$ act on a particle under the action of these forces, particle moves to point $3\hat{i} + \hat{j} + 4\hat{k}$ from the point $\hat{i} + 3\hat{j} - 2\hat{k}$ Find the work done.

39) A particle moves from a point $(0,1,-2)$ to $(-1,3,2)$ under the action of forces $(1,2,3)$, $(-1,2,3)$ and $(-1,2,-3)$, find the work done.

40) Forces $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$ are acting on a particle and the particle moves from $2\hat{i} + 3\hat{j} + \hat{k}$ to the point $5\hat{i} + 2\hat{j} + 3\hat{k}$ under these forces. find the work done

41) A particle moves from the point $3\hat{i} - 2\hat{j} + \hat{k}$ to the point $\hat{i} + 3\hat{j} - 4\hat{k}$ under the effect of constant forces $\hat{i} - \hat{j} + \hat{k}$ $\hat{i} + \hat{j} - 3\hat{k}$ and $4\hat{i} + 5\hat{j} - 6\hat{k}$ find the work done.

42) The constant forces $2\hat{i} + \hat{k} + \hat{j}$, $\hat{i} + \hat{j} + 2\hat{k}$, $2\hat{j} - 3\hat{k}$ acting on a particle displace it from the point $(5,3,2)$ to the point $(1,-1,2)$, find the work done.

43) A Force $\bar{F} = 2\hat{i} + \hat{j} + \hat{k}$ is acting at the point $(-3,2,1)$. Find the magnitude of the moment of force about the point $(2,1,2)$.

44) If $a = (1,-1,1)$, $b = (2,-1,1)$ and $c = (1,1,-2)$ then find $a \cdot (b+c)$.

45) Find x if $a = (2,3,-1)$ and $b = (x,-1,3)$ are perpendicular to each other.

46) The constant forces $(1,-1,1)$ $(1,1,-3)$ and $(4,5,-6)$ act on a particle. Under the action of these constant forces, particle moves from the point $(3,-2,1)$ to the point $(1,3,-4)$. Find the total work done by the forces.

UNIT-V

MARKS-10

MENSURATION

(MCQS-03MARKS,EXAMPLES-07 MARKS)

* MCQ'S

1) volume of cylinder with radius 'r' and height 'h' is ____.

2) The formula for volume of a sphere is ____.

3) If circumference of a circle is 10π cm then radius of a circle is ____.

4) Area of circle made from 4π cm long wire is ____ cm^2.

5) The area of a circle made from 8π cm long wire is ____ cm^2.

6) The area of rhombus whose diagonals are 30cm and 15 cm is ____ cm^2.

7) Area of rectangle with length 250cm and width 80cm is ____ sq.m.

8) If diameter of a circle is 14 cm. then area of circle is ____ sq.cm.

9) If Area of a square is 100 cm^2, then Perimeter of square = ____ cm.

10) Surface area of a cube of 5 cm length is ____ cm^2.

11) Volume of a cone whose radius is 4m and height is 12m is ____ m^3.

12) If the longest chord of a circle is 28 cm then its circumferences= ____.

13) If Diameter of a semi-sphere is 6 cm then its Volume = ____ cm^3.

14) 1 sq. meter = ____ sq. centimeter.

15) Volume of sphere having radius r = ____.

16) Diameter of a circle is 28 cm. Area of a circle is ____.

17) Area of a square having perimeter 2m is ____ cm^2.

*Example :-

1) If the circumference of a circle is equal to the area of a circle, find the radius of a circle.

2) A circle is made from 176 cm. long wire. Find the area of a circle.

3) The surface area of a sphere is 616 sq.cm. find the diameter of the sphere.

4) Two hemisphere of radius 5cm are attached at the end of cylinder of same radius. If height of cylinder is 16 cm, then find the surface area of shape.

5) How much papers required to prepare 20 cone shaped caps of radius 14cm of base and height 48 cm?

6) Find the volume of cylinder, whose radius is 5cm and height is 12 cm.

7) If the surface area of a spherical ball is 1256 sq.cm. Find the volume of the sphere. ($\pi = 3.14$)

8) Diameter of a circular common plot of a college is 42 m. To raise the height of surface up to 10 cm, how many m^3 clay is required?

9) How many spherical balls of radius 1cm can be made from cube of length 22 cm?

10) If measure of the three sides of a Triangle are 7cm 8cm and 9cm resp. Find area of Triangle.

11) How much milk can be contained in a cylindrical tank of 1.4 m radius & 3m height?

12) A metal solid cylinder has diameter 9cm and length 16 cm. How many small balls of 0.3 cm radius can be made from the cylinder?

13) Find the circumference of a circle having area 38.5 cm^2.

14) Length of one side of a rectangular plot is 35m and length of its one diagonal is 37 m. Find the area of a plot.

GTU QUESTION PAPER FORMAT

Seat No:_____ Enrolment No:_____

Gujarat Technological University

Diploma 1st Semester Examination-
Subject Code: 3300001
Subject Name: Basic Mathematics

Date: Time:
Total Marks:70

Instructions:

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible.(Scientific/Higher Version not allowed)
5. English version is authentic..

Q.1 Fill in the blanks using appropriate choice from the given options. 14

(1), (2) ,(3) from Logarithms
(4), (5) ,(6),(7) from Det and Matrices
(8), (9) ,(10),(11) from Trigonometry
(12), (13) ,(14), from Mensuration

Q.2

(A) Attempt any two 06

(1) From Logarithms
(2) From Mensuration
(3) From Mensuration

(B) Attempt any two 08

(1) From Logarithms
(2) From Logarithms
(3) From Mensuration

Q.3

(A) Attempt any two 06

(1) From Det and Matrix
(2) From Det and Matrix
(3) From Det and Matrix

(B) Attempt any two 08

(1) From Det and Matrix
(2) From Det and Matrix
(3) From Det and Matrix

Q.4

(A) Attempt any two — 06

(1) From Trigonometry

(2) From Trigonometry

(3) From Trigonometry

(B) Attempt any two — 08

(1) From Trigonometry

(2) From Trigonometry

(3) From Trigonometry

Q.5

(A) Attempt any two — 06

(1) From Vectors

(2) From Vectors

(3) From Vectors

(B) Attempt any two — 08

(1) From Vectors

(2) From Vectors

(3) From Vectors

********** *Best Wishes* **********