VPMP POLYTECHNIC, GANDHINAGAR

BASIC MATHEMATICS

CODE NO: - 3300001

SOLUTION BOOK

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UNIT-I - LOGARITHMS (10 MARKS)

$$x = a^{n} \iff \log_a x = n$$

$n \rightarrow$ Power, $a \rightarrow$ Base (Exponential Form)

$n \rightarrow$ logarithm, $a \rightarrow$ Base (Logarithmic Form)

Exponential Form   Logarithmic Form

* Rules of Logarithm :-

1. Logarithm of product :-

$$\log_a(xy) = \log_a x + \log_a y$$

e.g $\log_{10} 91 = \log_{10}(13 \times 7) = \log_{10} 13 + \log_{10} 7$

2. Logarithm of quotient

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

e.g $\log_{10}\left(\frac{7}{5}\right) = \log_{10} 7 - \log_{10} 5$

3. Logarithm of power

$$\log_a x^n = n.\log_a x, \quad n \in \mathbb{R}.$$

e.g $\log_{10} 9 = \log_{10} 3^2 = 2.\log_{10} 3$

4. Rule of change of Base

$$\log_y x = \frac{\log_a x}{\log_a y}$$

e.g $\log_7 15 = \dfrac{\log_{10} 15}{\log_{10} 7}$

* **Values of logarithm :-**

→ $\log_a 1 = 0, \ (a \neq 1)$

→ $\log_a a = 1$

→ $a^{\log_a y} = y.$

* **Question for 1 Marks :-**

1> $\log_2 64$

$= \log_2 2^6$

$= 6 \cdot \log_2 2$

$= 6 \cdot 1$

$= \boxed{6.}$

2> $\log_{10}(0.001)$

$= \log_{10} 10^{-3}$

$= -3 \log_{10} 10$

$= -3 \cdot 1$

$= \boxed{-3}$

3> $\log 32 \div \log 16$

$= \dfrac{\log 32}{\log 16}$

$= \dfrac{\log 2^5}{\log 2^4}$

$= \dfrac{5 \cdot \cancel{\log 2}}{4 \cdot \cancel{\log 2}} = \boxed{\dfrac{5}{4}}$

4> $\log_3\left(\frac{1}{27}\right)$

$= \log_3\left(\frac{1}{3^3}\right)$

$= \log_3 3^{-3}$

$= (-3) \log_3 3$

$= (-3) \cdot 1$

$= \boxed{-3}$

5> $\log 32 - \log 16$

$= \log\left(\frac{32}{16}\right)$

$= \log\left(\frac{2^5}{2^4}\right)$

$= \boxed{\log 2}$

6> $27^{\log_3 2}$

$= 3^{3\log_3 2}$

$= 3^{\log_3 2^3}$

$= 3^{\log_3 8}$

$= \boxed{8}$

7> $3^{\log_9 4} = 3^{\log_9 2^2} = 3^{2\log_9 2} = 9^{\log_9 2} = \boxed{2}$

8) $\log_7 49$

$= \log_7 7^2$

$= 2 \cdot \log_7 7$

$= 2 \cdot 1$

$= \boxed{2.}$

9). $\log_2 \left(\frac{1}{8}\right)$

$= \log_2 2^{-3}$

$= (-3) \log_2 2$

$= (-3) \cdot 1$

$= \boxed{-3}$

10) $\log_8 2$

$= \frac{\log 2}{\log 8}$

$= \frac{\log 2}{\log 2^3}$

$= \frac{\cancel{\log 2}}{3 \cdot \cancel{\log 2}} = \boxed{\frac{1}{3}}$

11) $\log 2 + \log \left(\frac{1}{2}\right)$

$= \log \left(\cancel{2} \cdot \frac{1}{\cancel{2}}\right)$

$= \log 1$

$= \boxed{0.}$

12) $\log_5 625$

$= \log_5 5^4$

$= 4 . \log_5 5$

$= \boxed{4.}$

13) $\log_{1/3} 9$

$= \frac{\log 9}{\log 1/3}$

$= \frac{\log 3^2}{\log 3^{-1}}$

$= \frac{2 \cancel{\log 3}}{(-1) \cancel{\log 3}}$

$= \boxed{-2}$

14) If $\log_2 x = 1$, then $x = ?$

$\log_2 x = 1$

$\therefore x = 2^1 = 2.$

$\therefore \boxed{x = 2}$

15) $\log 5 + \log 3$

$= \log (5 . 3)$

$= \log 15$

* Question for 3 Marks

1. Find the value of

$$\log\left(\frac{9}{14}\right) - \log\left(\frac{15}{16}\right) + \log\left(\frac{35}{24}\right)$$

$$= \log\left(\frac{9}{14}\right) + \log\left(\frac{35}{24}\right) - \log\left(\frac{15}{16}\right)$$

$$= \log\left[\frac{9/14 \times 35/24}{15/16}\right]$$

$$= \log\left[\frac{\cancel{9}^{3}}{\cancel{14}_{7}} \times \frac{\cancel{35}^{7}}{\cancel{24}_{8}} \times \frac{\cancel{16}^{8}}{\cancel{15}_{3}}\right]$$

$$= \log 1$$

$$= \boxed{0}$$

2. P.T. $\log\left(\frac{75}{16}\right) - 2\log\left(\frac{5}{9}\right) + \log\left(\frac{32}{243}\right) = \log 2.$

$$\text{L.H.S} = \log\left(\frac{75}{16}\right) + \log\left(\frac{32}{243}\right) - 2\log\left(\frac{5}{9}\right)$$

$$= \log\left(\frac{75}{\cancel{16}} \times \frac{\cancel{32}^{2}}{243}\right) - \log\left(\frac{5}{9}\right)^2.$$

$$= \log\left(\frac{75 \times 2}{243}\right) - \log\frac{25}{81}$$

$$= \log\left(\frac{\cancel{75}^{25} \times 2}{\cancel{243}_{3}} \times \frac{\cancel{81}}{\cancel{25}}\right)$$

$$= \log 2$$

$$= \text{R.H.S.}$$

3> Simplify $\log\left(\frac{450}{32}\right) + \log\left(\frac{25}{128}\right) + \log\left(\frac{64}{225}\right) + \log\left(\frac{32}{25}\right)$

$$= \log\left[\frac{\cancel{450}^{225}}{\cancel{32}} \times \frac{\cancel{25}}{\cancel{128}_{2}} \times \frac{64}{\cancel{225}} \times \frac{\cancel{32}}{\cancel{25}}\right]$$

$$= \log 1$$

$$= \boxed{0}$$

4> P. T. $\log(x + \sqrt{x^2 - 1}) + \log(x - \sqrt{x^2 - 1}) = 0.$

$$\text{L.H.S} = \log(x + \sqrt{x^2 - 1}) + \log(x - \sqrt{x^2 - 1})$$

$$= \log\left[(x + \sqrt{x^2 - 1}) \cdot (x - \sqrt{x^2 - 1})\right]$$

$$= \log\left[x^2 - (\sqrt{x^2 - 1})^2\right]$$

$$= \log(x^2 - (x^2 - 1))$$

$$= \log(\cancel{x^2} - \cancel{x^2} + 1)$$

$$= \log 1$$

$$= 0$$

$$= \text{R.H.S.}$$

5>. P. T. $\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ca}\right) + \log\left(\frac{c^2}{ab}\right) = 0$

$$\text{L.H.S} = \log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ca}\right) + \log\left(\frac{c^2}{ab}\right)$$

$$= \log\left[\frac{\cancel{a^2}}{\cancel{bc}} \times \frac{\cancel{b^2}}{\cancel{ca}} \times \frac{\cancel{c^2}}{\cancel{ab}}\right]$$

$$= \log 1 \quad = 0 \quad = \text{R.H.S}$$

9) P.T. $\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2$

L.H.S $= \frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24}$

$= \frac{1}{\frac{\log 24}{\log 6}} + \frac{1}{\frac{\log 24}{\log 12}} + \frac{1}{\frac{\log 24}{\log 8}}$

$= \frac{\log 6}{\log 24} + \frac{\log 12}{\log 24} + \frac{\log 8}{\log 24}$

$= \frac{\log 6 + \log 12 + \log 8}{\log 24}$

$= \frac{\log(6 \cdot 12 \cdot 8)}{\log 24}$

$= \frac{\log (24)^2}{\log 24}$

$= \frac{2 \cdot \log 24}{\log 24}$

$= 2$

$=$ R.H.S.

10) P.T. $\log_y x^2 \times \log_z y^3 \times \log_x z^4 = 24$

L.H.S $= \log_y x^2 \times \log_z y^3 \times \log_x z^4$

$= 2 \cdot \log_y x \times 3 \cdot \log_z y \times 4 \cdot \log_x z$

$= (2 \cdot 3 \cdot 4) \left[\frac{\log x}{\log y} \times \frac{\log y}{\log z} \times \frac{\log z}{\log x} \right]$

$= 24$ $=$ R.H.S.

11> P.T. $\dfrac{1}{\log_2 6} + \dfrac{1}{\log_3 6} = 1.$

$$\text{L.H.S} = \frac{1}{\log_2 6} + \frac{1}{\log_3 6}$$

$$= \frac{1}{\frac{\log 6}{\log 2}} + \frac{1}{\frac{\log 6}{\log 3}}$$

$$= \frac{\log 2}{\log 6} + \frac{\log 3}{\log 6}$$

$$= \frac{\log 2 + \log 3}{\log 6}$$

$$= \frac{\log(2\cdot 3)}{\log 6} = \frac{\log 6}{\log 6} = 1 = \text{R.H.S.}$$

12> P.T. $\log_x y \times \log_y z \times \log_z \omega = \log_x \omega$

$$\text{L.H.S} = \log_x y \times \log_y z \times \log_z \omega$$

$$= \frac{\cancel{\log y}}{\log x} \times \frac{\cancel{\log z}}{\cancel{\log y}} \times \frac{\log \omega}{\cancel{\log z}}$$

$$= \frac{\log \omega}{\log x} = \log_x \omega = \text{R.H.S.}$$

13> If $\log_a x^3 - \log_a 25 = \log_a x$ then find value of x.

$$\therefore \log_a x^3 - \log_a 25 = \log_a x$$

$$\therefore \log_a \left(\frac{x^3}{25}\right) = \log_a x$$

$\therefore \frac{x^{3}}{25} = x$

$\therefore x^2 = 25$

$\therefore \boxed{x = \pm 5}$

but $x \neq -5$ because logarithm of negative number can't be defined.

$\therefore \boxed{x = 5}$

14) If $\frac{4 \log 3 \times \log x}{\log 9} = \log 27$, then find the value of x.

$\rightarrow$ $\frac{4 \log 3 \times \log x}{\log 9} = \log 27$

$\therefore \frac{(2 \times 2) \log 3 \times \log x}{\log 9} = \log 27$

$\therefore \frac{2 \log 3^2 \times \log x}{\log 9} = \log 27$

$\therefore \frac{2 \cancel{\log 9} \times \log x}{\log 9} = \log 27$

$\therefore 2 \log x = \log 27$

$\therefore \log x^2 = \log 27$

$\therefore x^2 = 27$

$\therefore \boxed{x = 3\sqrt{3}}$

15) If $\log_2 x = 2$ and $\log_x y = 2$ then find the value of y.

$\log_2 x = 2$, $\log_x y = 2$

$\therefore x = 2^2 = 4$ $\therefore \log_4 y = 2$

$\therefore \boxed{x = 4}$ $\therefore y = 4^2$

$\therefore \boxed{y = 16}$

* <u>Question for 4 Marks</u>

1). P. T. $\dfrac{1}{\log_a abc} + \dfrac{1}{\log_b abc} + \dfrac{1}{\log_c abc} = 1$.

$$\text{L.H.S} = \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$$

$$= \frac{1}{\frac{\log abc}{\log a}} + \frac{1}{\frac{\log abc}{\log b}} + \frac{1}{\frac{\log abc}{\log c}}$$

$$= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$$

$$= \frac{\log a + \log b + \log c}{\log abc}$$

$$= \frac{\log abc}{\log abc} = 1 = \text{R.H.S.}$$

2) P. T. $\dfrac{1}{\log_{bc} p} + \dfrac{1}{\log_{ca} p} + \dfrac{1}{\log_{ab} p} = 2 \cdot \log_p abc$

$$\text{L.H.S} = \frac{1}{\log_{bc} p} + \frac{1}{\log_{ca} p} + \frac{1}{\log_{ab} p}$$

$$= \frac{1}{\frac{\log p}{\log bc}} + \frac{1}{\frac{\log p}{\log ca}} + \frac{1}{\frac{\log p}{\log ab}}$$

$$= \frac{\log bc}{\log p} + \frac{\log ca}{\log p} + \frac{\log ab}{\log p}$$

$$= \frac{\log bc + \log ca + \log ab}{\log p}$$

$$= \frac{\log (bc \cdot ca \cdot ab)}{\log p}$$

$$= \frac{\log (a^2 b^2 c^2)}{\log p}$$

$$= \frac{\log (abc)^2}{\log p} = \frac{2 \cdot \log abc}{\log p} = 2 \cdot \log_p abc$$

3) If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then P.T. $a = b$.

$\rightarrow \log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\therefore \log\left(\frac{a+b}{2}\right) = \frac{1}{2}\log(ab)$

$\therefore 2\log\left(\frac{a+b}{2}\right) = \log(ab)$

$\therefore \log\left(\frac{a+b}{2}\right)^2 = \log(ab)$

$\therefore \frac{a^2 + 2ab + b^2}{4} = ab$

$\therefore a^2 + 2ab + b^2 = 4ab$

$\therefore a^2 + 2ab - 4ab + b^2 = 0$

$\therefore a^2 - 2ab + b^2 = 0.$

$\therefore (a-b)^2 = 0$

$\therefore a-b=0$

$\therefore \boxed{a=b.}$

4> P. T. $\log_a p + \log_{a^2} p^2 + \log_{a^3} p^3 + \log_{a^4} p^4 = 4 \cdot \log_a p$

L.H.S :- $\log_a p + \log_{a^2} p^2 + \log_{a^3} p^3 + \log_{a^4} p^4$

$$= \frac{\log p}{\log a} + \frac{\log p^2}{\log a^2} + \frac{\log p^3}{\log a^3} + \frac{\log p^4}{\log a^4}$$

$$= \frac{\log p}{\log a} + \frac{\cancel{2} \cdot \log p}{\cancel{2} \cdot \log a} + \frac{\cancel{3} \cdot \log p}{\cancel{3} \cdot \log a} + \frac{\cancel{4} \cdot \log p}{\cancel{4} \cdot \log a}$$

$$= \frac{\log p}{\log a} + \frac{\log p}{\log a} + \frac{\log p}{\log a} + \frac{\log p}{\log a}$$

$$= 4 \cdot \frac{\log p}{\log a}$$

$= 4 \log_a p \quad$ = R.H.S.

5> P. T. $\log_x \left(\frac{a-b}{b-c}\right) + \log_x \left(\frac{b-c}{c-a}\right) + \log_x \left(\frac{c-a}{a-b}\right) = 0$

L.H.S = $\log_x \left(\frac{a-b}{b-c}\right) + \log_x \left(\frac{b-c}{c-a}\right) + \log_x \left(\frac{c-a}{a-b}\right)$

$$= \log_x \left[\frac{\cancel{a-b}}{\cancel{b-c}} \times \frac{\cancel{b-c}}{\cancel{c-a}} \times \frac{\cancel{c-a}}{\cancel{a-b}} \right]$$

$= \log_x 1$

$= 0$

= R.H.S.

6) P.T. $\log_{\sqrt{q}} p^2 \cdot \log_{\sqrt{r}} q^2 \cdot \log_{\sqrt{p}} r^2 = 64$

$$L.H.S = \log_{\sqrt{q}} p^2 \cdot \log_{\sqrt{r}} q^2 \cdot \log_{\sqrt{p}} r^2$$

$$= \frac{\log p^2}{\log\sqrt{q}} \cdot \frac{\log q^2}{\log\sqrt{r}} \cdot \frac{\log r^2}{\log\sqrt{p}}$$

$$= \frac{\log p^2}{\log q^{1/2}} \cdot \frac{\log q^2}{\log r^{1/2}} \cdot \frac{\log r^2}{\log p^{1/2}}$$

$$= \frac{2.\cancel{\log p}}{\frac{1}{2}\cancel{\log q}} \cdot \frac{2\cancel{\log q}}{\frac{1}{2}\cancel{\log r}} \cdot \frac{2.\cancel{\log r}}{\frac{1}{2}\cancel{\log p}}$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$= 64 = R.H.S$$

7) P.T. $\frac{1}{\log_x yz + 1} + \frac{1}{\log_y zx + 1} + \frac{1}{\log_z xy + 1} = 1.$

$$L.H.S = \frac{1}{\log_x yz + 1} + \frac{1}{\log_y zx + 1} + \frac{1}{\log_z xy + 1}$$

$$= \frac{1}{\log_x yz + \log_x x} + \frac{1}{\log_y zx + \log_y y} + \frac{1}{\log_z xy + \log_z z}$$

$$= \frac{1}{\log_x xyz} + \frac{1}{\log_y xyz} + \frac{1}{\log_z xyz}$$

$$= \frac{\log x}{\log xyz} + \frac{\log y}{\log xyz} + \frac{\log z}{\log xyz}$$

$$= \frac{\log x + \log y + \log z}{\log xyz}$$

$$= \frac{\cancel{\log xyz}}{\cancel{\log xyz}} = 1 = R.H.S$$

8> Solve the following eq^n.

$\log(2x+1) + \log(3x-1) = 0.$

$\therefore \log_{10}((2x+1)\cdot(3x-1)) = 0$

$\therefore (2x+1)\cdot(3x-1) = 10^0$

$\therefore 6x^2 - 2x + 3x - 1 = 1$

$\therefore 6x^2 + x - 1 - 1 = 0$

$\therefore 6x^2 + x - 2 = 0.$

$\therefore \underline{6x^2 + 4x} \underline{- 3x - 2} = 0$

$\therefore 2x(3x+2) - 1(3x+2) = 0$

$\therefore (3x+2)(2x-1) = 0.$

$\therefore 3x + 2 = 0$ & $2x - 1 = 0.$

$\therefore 3x = -2$ $\therefore 2x = 1$

$\therefore x = -2/3$ $\therefore x = 1/2$

But $x \neq -2/3$ because logarithm of negative number can't be defined.

$\therefore \boxed{x = 1/2}$

9> If $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$, then P.T. $x^2 + y^2 = 7xy$.

$\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$

$\therefore 2\cdot\log\left(\frac{x+y}{3}\right) = \log x + \log y$

$\therefore \log\left(\frac{x+y}{3}\right)^2 = \log(xy)$

$\therefore \left(\frac{x+y}{3}\right)^2 = xy$

$$\therefore \frac{(x+y)^2}{9} = xy$$

$$\therefore x^2 + 2xy + y^2 = 9xy$$

$$\therefore x^2 + y^2 = 9xy - 2xy$$

$$\therefore x^2 + y^2 = 7xy.$$

10). If $\dfrac{2\log_5^x + \log_5^9}{\log_5^{3x}} = \log_5^x$ then find the value of x.

$$\therefore \frac{2\log_5^x + \log_5^{3^2}}{\log_5^{3x}} = \log_5^x$$

$$\therefore \frac{2\log_5^x + 2.\log_5^3}{\log_5^{3x}} = \log_5^x$$

$$\therefore \frac{2(\log_5^x + \log_5^3)}{\log_5^{3x}} = \log_5^x$$

$$\therefore \frac{2(\cancel{\log_5^{3x}})}{\cancel{\log_5^{3x}}} = \log_5^x$$

$$\therefore \log_5^x = 2$$

$$\therefore x = 5^2$$

$$\therefore \boxed{x = 25}$$