UNIT-II

MARKS-18

DETERMINAT

&

MATRICES

(MCQS-04MARKS,EXAMPLES-14MARKS)

UNIT-II - DETERMINANT & MATRICES
(18 MARKS IN GTU)

* DETERMINANT :-

* 2X2 Determinant

column

Row

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

* 3X3 Determinant

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

* MATRICES

Ex

$$\begin{bmatrix} 1 & 5 & -3 \\ 2 & 6 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 8 \\ 6 & -2 \end{bmatrix}, \begin{bmatrix} 5 & 6 & 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

* TYPES OF MATRICES

* Row - Matrix

Ex

$$\begin{bmatrix} 4 & 3 & 9 \end{bmatrix}$$

* Column Matrix

Ex

$$\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$$

* Trnspose Matrix $(A')' = A$ or $(A^T)^T = A$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & -3 \\ 4 & 0 & 1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 5 & 4 \\ 3 & 6 & 0 \\ 4 & -3 & 1 \end{bmatrix}$$

* **Null Matrix**

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2}, \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3}$$

* **Square Matrix**

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2}. \quad \begin{bmatrix} 3 & -2 & 5 \\ 4 & 1 & 0 \\ 7 & 0 & -8 \end{bmatrix}_{3\times3}$$

* **Diagonal Matrix**

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

* **Unit - Matrix** or **Identity Matrix**

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* **Scalar Matrix**

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

* **Singular Matrix** : $\boxed{|A| = 0}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 3 & 6 & 9 \end{bmatrix} \Rightarrow |A| = 0$$ then A is singular Matrix.

* Adjoint of a matrix.

→ 2×2

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \text{Adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

→ 3 × 3.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Find co-factor of each element of matrix A.
Then taking Transpose of that.
So, we get Adjoint of matrix.

* Inverse of Matrix

→ $A^{-1} = \dfrac{\text{Adj } A}{|A|}$.

First Find determinant of given matrix
If $|A| = 0$ then A^{-1} does not exists.
If $|A| \neq 0$ then A^{-1} exists.
Then we can find co-factor of each element of the given Matrix.
So, we can find inverse of the matrix using Formula.

$$\boxed{A^{-1} = \frac{\text{Adj } A}{|A|}}$$

* Question for 1 Marks

1> Evaluate $\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$

$\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} = 4 - 6 = -2$

2> If $\begin{vmatrix} x & 1 \\ 4 & 2 \end{vmatrix} = 0$ then find x.

$\therefore 2x - 4 = 0$

$\therefore 2x = 4$

$\therefore \boxed{x = 2}$

3> $\begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$

$= \sin^2\theta + \cos^2\theta = 1$

4> $\begin{vmatrix} 1 & 7 & -3 \\ -4 & 6 & 2 \\ 2 & -5 & 3 \end{vmatrix}$

$= 1 \begin{vmatrix} 6 & 2 \\ -5 & 3 \end{vmatrix} - 7 \begin{vmatrix} -4 & 2 \\ 2 & 3 \end{vmatrix} + (-3) \begin{vmatrix} -4 & 6 \\ 2 & -5 \end{vmatrix}$

$= 1(18 + 10) - 7(-12 - 4) - 3(20 - 12)$

$= 1(28) - 7(-16) - 3(8)$

$= 28 + 112 - 24$

$= 116$

5) $\begin{vmatrix} 3 & 4 & 5 \\ 0 & 3 & 2 \\ 2 & -4 & -5 \end{vmatrix}$

$= 3\begin{vmatrix} 3 & 2 \\ -4 & -5 \end{vmatrix} - 4\begin{vmatrix} 0 & 2 \\ 2 & -5 \end{vmatrix} + 5\begin{vmatrix} 0 & 3 \\ 2 & -4 \end{vmatrix}$

$= 3(-15+8) - 4(0-4) + 5(0-6)$

$= 3(-7) - 4(-4) + 5(-6)$

$= -21 + 16 - 30$

$= -51 + 16$

$= \boxed{-35}$

6) $\begin{vmatrix} 3 & 2 & 1 \\ -1 & 2 & 6 \\ 3 & 0 & 5 \end{vmatrix}$

$= 3\begin{vmatrix} 2 & 6 \\ 0 & 5 \end{vmatrix} - 2\begin{vmatrix} -1 & 6 \\ 3 & 5 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 3 & 0 \end{vmatrix}$

$= 3(10-0) - 2(-5-18) + 1(0-6)$

$= 3(10) - 2(-23) + 1(-6)$

$= 30 + 46 - 6$

$= 30 + 40$

$= \boxed{70}$

7) $\begin{vmatrix} x-2 & 3 \\ 0 & x+2 \end{vmatrix} = 0$ then find x.

$\therefore (x-2)(x+2) - 0 = 0$

$\therefore x^2 - 4 = 0$

$\therefore x^2 = 4$

$\therefore \boxed{x = \pm 2}$

8) $\begin{vmatrix} 5 & x+3 \\ x+5 & 5 \end{vmatrix} = 0$ then find x.

$\therefore 25 - (x+3)(x+5) = 0$

$\therefore 25 - [x^2+5x+3x+15] = 0$

$\therefore 25 - (x^2+8x+15) = 0$

$\therefore 25 - x^2 - 8x - 15 = 0$

$\therefore -x^2 - 8x + 10 = 0$

$\therefore x^2 + 8x - 10 = 0$

9) $\begin{bmatrix} 1 & 5 \\ -3 & 2 \\ -5 & 4 \end{bmatrix} - \begin{bmatrix} 0 & -4 \\ 3 & -2 \\ 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1-0 & 5+4 \\ -3-3 & 2+2 \\ -5-4 & 4-3 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ -6 & 4 \\ -9 & 1 \end{bmatrix}$

10) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 2-1 \\ 3+1 & 4+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 7 \end{bmatrix}$

11) If $A = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$ then find A^2.

$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 1+10 & 2+12 \\ 5+30 & 10+36 \end{bmatrix}$

$= \begin{bmatrix} 11 & 14 \\ 35 & 46 \end{bmatrix}$

12) $\begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5+4 & 6+2 \\ 25+12 & 30+6 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 37 & 36 \end{bmatrix}$

13) Order of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 4 \end{bmatrix} = \underline{3 \times 2.}$

14) order of $\begin{bmatrix} 2 & -1 & 5 & 3 & 2 \\ 7 & 3 & 4 & 2 & 1 \\ 5 & 2 & 1 & -2 & 3 \\ 1 & 4 & 8 & 9 & -3 \end{bmatrix} = \underline{4 \times 5}$

15) $\begin{bmatrix} 1 & 3 & 4 & 9 & 3 \\ -2 & 3 & 0 & 4 & 2 \\ 5 & 6 & 8 & 2 & 7 \\ 6 & 4 & 3 & 2 & 0 \end{bmatrix}^T \Rightarrow \begin{bmatrix} 1 & -2 & 5 & 6 \\ 3 & 3 & 6 & 4 \\ 4 & 0 & 8 & 3 \\ 9 & 4 & 2 & 2 \\ 3 & 2 & 7 & 0 \end{bmatrix}$

16) $\begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1+6+2 \end{bmatrix} = \begin{bmatrix} 9 \end{bmatrix}$

17) Find Adj of $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$

18) Find Adj of $\begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix}$$

* Question for 3 Marks :-

* If $\begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$ find x.

$$\therefore \begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$$

$$\therefore (x-2)\left[4x - 0\right] - 2\left[-4+4\right] + 2\left[0 - 2x\right] = 0$$

$$\therefore (x-2)(4x) - 2(0) + 2(-2x) = 0$$

$$\therefore 4x^2 - 8x - 4x = 0$$

$$\therefore 4x^2 - 12x = 0$$

$$\therefore 4x(x-3) = 0$$

$$\therefore 4x = 0 \quad \& \quad x - 3 = 0$$

$$\therefore \boxed{x = 0} \quad \& \quad \boxed{x = 3}$$

* Let $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & 5 \\ 5 & 4 & 3 \\ 3 & 5 & 4 \end{bmatrix}$ and

$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 4 & 5 & 6 \end{bmatrix}$ find $3A + 2B - C$

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$\rightarrow$ $3A = \begin{bmatrix} 3 & 9 & 15 \\ -3 & 0 & 6 \\ 12 & 9 & 18 \end{bmatrix}$, $2B = \begin{bmatrix} 6 & 8 & 10 \\ 10 & 8 & 6 \\ 6 & 10 & 8 \end{bmatrix}$

Now, $3A + 2B - C$

$$= \begin{bmatrix} 3 & 9 & 15 \\ -3 & 0 & 6 \\ 12 & 9 & 18 \end{bmatrix} + \begin{bmatrix} 6 & 8 & 10 \\ 10 & 8 & 6 \\ 6 & 10 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 3+6-1 & 9+8-2 & 15+10-1 \\ -3+10-3 & 0+8-2 & 6+6-2 \\ 12+6-4 & 9+10-5 & 18+8-6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 15 & 24 \\ 4 & 6 & 10 \\ 14 & 14 & 20 \end{bmatrix}$$

* Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & 1 \\ 1 & 7 & 5 \end{bmatrix}$, find AB.

$\rightarrow$ $AB = \begin{bmatrix} 1 & -1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & 1 \\ 1 & 7 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 1-4+1 & 2-2+7 & 1-1+5 \\ 3+8+1 & 6+4+7 & 3+2+5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 7 & 5 \\ 12 & 17 & 10 \end{bmatrix}$$

* If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ then find AB and BA, Is $AB = BA$?

→ $AB = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1-3 & 2-2 \\ -1+3 & -2+2 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 \\ 2 & 0 \end{bmatrix}$ —— (*)

→ $BA = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & -1+2 \\ 3-2 & -3+2 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ —— (**)

From (*) & (**) we have $\boxed{AB \neq BA}$

* $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}$ then P.T. $A'B' = (BA)'$

→ $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$B = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$

$$\rightarrow A'B' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6 & 1+2 \\ -6+12 & 3+4 \end{bmatrix}$$

$$A'B' = \begin{bmatrix} 4 & 3 \\ 6 & 7 \end{bmatrix} \quad \text{—} \quad (1)$$

$$\rightarrow BA = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6 & -6+12 \\ 1+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 3 & 7 \end{bmatrix}.$$

$$(BA)' = \begin{bmatrix} 4 & 3 \\ 6 & 7 \end{bmatrix} \quad \text{—} \quad (2)$$

From (1) & (2) we get $A'B' = (BA)'$

* If $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & 1 & 2 \end{bmatrix}$ then find A^2.

$$A \cdot A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+0 & 0+0+0 & 0+0+0 \\ 2+3+0 & 0+9+0 & 0+0+0 \\ 8+1+8 & 0+3+2 & 0+0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 5 & 9 & 0 \\ 17 & 5 & 4 \end{bmatrix}$$

* If $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$ then find $A^2 - 2A + 3I$.

→ $A^2 = A \cdot A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4+6 & 6+3 \\ 4+2 & 6+1 \end{bmatrix}$

$= \begin{bmatrix} 10 & 9 \\ 6 & 7 \end{bmatrix}$

→ $2A = 2 \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 4 & 2 \end{bmatrix}$

→ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

→ $A^2 - 2A + 3I = \begin{bmatrix} 10 & 9 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 10-4+3 & 9-6+0 \\ 6-4+0 & 7-2+3 \end{bmatrix}$

$= \begin{bmatrix} 9 & 3 \\ 2 & 8 \end{bmatrix}$

* If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 5A + 7I = 0$.

→ $A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\rightarrow +5A = +5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$\rightarrow I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$L.H.S = A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0 = R.H.S$

* If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then P.T. $\text{Adj}\,A = A$.

$\rightarrow$ First we have to find co-factor of matrix A.

$\rightarrow$ co-factor of $(-4) = (-1)^{1+1}\begin{vmatrix} 0 & 1 \\ 4 & 3 \end{vmatrix} = 0-4 = -4$

$\rightarrow$ co-factor of $(-3) = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = -(3-4) = -(-1) = 1$

$\rightarrow$ co-factor of $(-3) = (-1)^{1+3}\begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = 4-0 = 4.$

$\rightarrow$ co-factor of $1 = (-1)^{2+1}\begin{vmatrix}-3 & -3\\ 4 & 3\end{vmatrix} = -(-9+12) = -3$

$\rightarrow$ co-factor of $0 = (-1)^{2+2}\begin{vmatrix}-4 & -3\\ 4 & 3\end{vmatrix} = -12+12 = 0$

$\rightarrow$ co-factor of $1 = (-1)^{2+3}\begin{vmatrix}-4 & -3\\ 4 & 4\end{vmatrix} = -(-16+12) = -(-4) = 4$

$\rightarrow$ co-factor of $4 = (-1)^{3+1}\begin{vmatrix}-3 & -3\\ 0 & 1\end{vmatrix} = -3-0 = -3$

$\rightarrow$ co-factor of $4 = (-1)^{3+2}\begin{vmatrix}-4 & -3\\ 1 & 1\end{vmatrix} = -(-4+3) = 1$

$\rightarrow$ co-factor of $3 = (-1)^{3+3}\begin{vmatrix}-4 & -3\\ 1 & 0\end{vmatrix} = 0+3 = 3$

$$\text{Adj } A = \begin{bmatrix}-4 & -3 & -3\\ 1 & 0 & 1\\ 4 & 4 & 3\end{bmatrix}$$

Thus Adj $A = A$.

* If $A = \begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ then show that $A \cdot A^{-1} = I$.

$\rightarrow$ $|A| = \begin{vmatrix}1 & -1\\ 2 & 3\end{vmatrix} = 3-(-2) = 3+2 = 5 \neq 0$

$\therefore \boxed{|A| = 5 \neq 0}$

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So, A^{-1} is exist.

Now, $A^{-1} = \dfrac{\text{Adj } A}{|A|}$

$$= \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

L.H.S $= A \cdot A^{-1}$

$$= \frac{1}{5}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 3+2 & 1-1 \\ 6-6 & 2+3 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= I. \quad =$ R.H.S.

* Find the inverse of $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{vmatrix} = 3\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 4 & -1 \\ 5 & 1 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 \\ 5 & 0 \end{vmatrix}$$

$\therefore |A| = 3(1-0) + 1(4+5) + 2(0-5)$

$= 3(1) + 1(9) + 2(-5)$

$= 3 + 9 - 10 = 12 - 10 = 2 \neq 0$

$\therefore |A| = 2 \neq 0$

so, A^{-1} exists. For that we have to find.

Adjoint of matrix.

$\rightarrow$ co-factor of $3 = (-1)^{1+1} \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1-0 = 1$

$\rightarrow$ co-factor of $(-1) = (-1)^{1+2} \begin{vmatrix} 4 & -1 \\ 5 & 1 \end{vmatrix} = -(4+5) = -9$

$\rightarrow$ co-factor of $2 = (-1)^{1+3} \begin{vmatrix} 4 & 1 \\ 5 & 0 \end{vmatrix} = 0-5 = -5$

$\rightarrow$ co-factor of $4 = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$

$\rightarrow$ co-factor of $1 = (-1)^{2+2} \begin{vmatrix} 3 & 2 \\ 5 & 1 \end{vmatrix} = 3-10 = -7$

$\rightarrow$ co-factor of $(-1) = (-1)^{2+3} \begin{vmatrix} 3 & -1 \\ 5 & 0 \end{vmatrix} = -(0+5) = -5$

$\rightarrow$ co-factor of $5 = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1-2 = -1$

$\rightarrow$ co-factor of $0 = (-1)^{3+2} \begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} = -(-3-8) = 11$

$\rightarrow$ co-factor of $1 = (-1)^{3+3} \begin{vmatrix} 3 & -1 \\ 4 & 1 \end{vmatrix} = 3+4 = 7.$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}$$

Now, $A^{-1} = \frac{\text{Adj } A}{|A|}$

$$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}$$

* Find the inverse of $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$\rightarrow$ $|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} - (-3)\begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} + 4\begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix}$

$= 3(-3+4) + 3(2-0) + 4(-2+0)$

$= 3(1) + 3(2) + 4(-2)$

$= 3 + 6 - 8$

$= 9 - 8 = 1 \neq 0$

$\therefore \boxed{|A| = 1 \neq 0}$

So, A^{-1} exists.

Now, we have to find Adjoint of matrix A.

$\rightarrow$ Co-factor of $3 = (-1)^{1+1}\begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -3 + 4 = 1.$

$\rightarrow$ Co-factor of $(-3) = (-1)^{1+2}\begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = -(2-0) = -2$

$\rightarrow$ Co-factor of $4 = (-1)^{1+3}\begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = -2+0 = -2$

$\rightarrow$ co-factor of $2 = (-1)^{2+1} \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -(-3+4) = -1$

$\rightarrow$ co-factor of $(-3) = (-1)^{2+2} \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

$\rightarrow$ co-factor of $4 = (-1)^{2+3} \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = -(-9+6) = -(-3) = 3$

$\rightarrow$ co-factor of $0 = (-1)^{3+1} \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} = -12 + 12 = 0$

$\rightarrow$ co-factor of $(-1) = (-1)^{3+2} \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = -(12-8) = -4.$

$\rightarrow$ co-factor of $1 = (-1)^{3+3} \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = -9+6 = -3.$

$$\text{Adj } A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{\text{Adj } A}{|A|}$

$$= 1 \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

* If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ then find Matrix B, such that $AB = BA = I$.

→ Here we have $AB = I = BA$.

Consider $AB = I$.

$\therefore B = A^{-1} \cdot I$. —— (1)

Now,

$$|A| = \begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1 \neq 0$$

$$\therefore \boxed{|A| = 1 \neq 0}$$

So, A^{-1} exists.

$$\text{Adj } A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A.$$

$$\therefore A^{-1} = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

Now, From (1)

$\therefore B = A^{-1} \cdot I$.

$$\therefore B = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

* Solve $3x + y = 9$ and $2x - 3y = -5$ using Matrices.

$$\begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$\therefore AX = B.$, where $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$

and $B = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$\therefore X = A^{-1} \cdot B.$ —— (1).

For that

$$|A| = \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} = -9 - 2 = -11 \neq 0$$

$\therefore |A| = -11 \neq 0.$

Thus A^{-1} exists.

Now, $A^{-1} = \dfrac{\text{Adj } A}{|A|}$

$$\therefore A^{-1} = \frac{1}{-11} \begin{bmatrix} -3 & -1 \\ -2 & 3 \end{bmatrix}$$

Now, From (1) we have.

$X = A^{-1} \cdot B$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{(-11)} \begin{bmatrix} -3 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$= \frac{1}{(-11)} \begin{bmatrix} -27 + 5 \\ -18 - 15 \end{bmatrix} = \frac{1}{(-11)} \begin{bmatrix} -22 \\ -33 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow \boxed{x = 2} \ \& \ \boxed{x = 3}$$

* Solve $2x+3y=1$ and $y-4x=2$ using Matrices.

→ $2x+3y=1$

$-4x+y=2$

$$\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$AX = B$, where $A = \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$

and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\therefore X = A^{-1} \cdot B$ — (1)

For that $|A| = \begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix} = 2+12 = 14 \neq 0.$

$\therefore \boxed{|A| = 14 \neq 0}$

So, A^{-1} exists.

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{1}{14}\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}.$$

Now, From (1). $X = A^{-1} \cdot B.$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} 1-6 \\ 4+4 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} -5 \\ 8 \end{bmatrix} = \begin{bmatrix} -5/14 \\ 8/14 \end{bmatrix}$$

$$\therefore \boxed{x = \frac{-5}{14}} \quad \& \quad \boxed{y = \frac{4}{7}}$$

* **Question for 4 Marks**

* If $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix}$ then find $(A+B)^{-1}$.

→ $A+B = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 4 \end{bmatrix}$

→ Now, $|A+B| = \begin{vmatrix} 2 & 0 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 4 \end{vmatrix}$

$= 2(8-1) - 0(12-2) + 3(3-4)$

$= 2(7) - 0 + 3(-1)$

$= 14 - 3$

$= 11 \neq 0.$

$\therefore \boxed{|A+B| = 11 \neq 0}$

→ Now for A^{-1} we have to find Adjoint of A+B Matrix

→ Co-factor of $2 = (-1)^{1+1} \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} = 8-1 = 7.$

→ Co-factor of $0 = (-1)^{1+2} \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = -(12-2) = -10$

→ Co-factor of $3 = (-1)^{1+3} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = 3-4 = -1.$

$\rightarrow$ co-factor of $3 = (-1)^{2+1}\begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix} = -(0-3) = 3$

$\rightarrow$ co-factor of $2 = (-1)^{2+2}\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 8-6 = 2.$

$\rightarrow$ co-factor of $1 = (-1)^{2+3}\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = -(2-0) = -2.$

$\rightarrow$ co-factor of $2 = (-1)^{3+1}\begin{vmatrix} 0 & 3 \\ 2 & 1 \end{vmatrix} = 0-6 = -6$

$\rightarrow$ co-factor of $1 = (-1)^{3+2}\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = -(2-9) = 7$

$\rightarrow$ co-factor of $4 = (-1)^{3+3}\begin{vmatrix} 2 & 0 \\ 3 & 2 \end{vmatrix} = 4-0 = 4.$

$$\text{Adj } A+B = \begin{bmatrix} 7 & 3 & -6 \\ -10 & 2 & 7 \\ -1 & -2 & 4 \end{bmatrix}$$

$$(A+B)^{-1} = \frac{\text{Adj}(A+B)}{|A+B|} = \frac{1}{11}\begin{bmatrix} 7 & 3 & -6 \\ -10 & 2 & 7 \\ -1 & -2 & 4 \end{bmatrix}$$

* If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ show that $A \cdot A^{-1} = I$

$\rightarrow$ $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix}$

$= 1(4-3) - 2(6-3) + 1(3-2)$

$= 1(1) - 2(3) + 1(1)$

$= 1 - 6 + 1$

$= -4 \neq 0$

$\therefore \boxed{|A| = -4 \neq 0}$

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$\therefore$ so A^{-1} exists.

For that we have to find Adjoint of A.

→ co-factor of $1 = (-1)^{1+1} \cdot \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1.$

→ co-factor of $2 = (-1)^{1+2} \begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix} = -(6-3) = -3$

→ co-factor of $1 = (-1)^{1+3} \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1.$

→ co-factor of $3 = (-1)^{2+1} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -(4-1) = -3$

→ co-factor of $\cancel{1}2 = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1.$

→ co-factor of $3 = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -(1-2) = 1$

→ co-factor of $1 = (-1)^{3+1} \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 6 - 2 = 4$

→ co-factor of $1 = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = -(3-3) = 0.$

→ co-factor of $2 = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 2 - 6 = -4$

$\text{Adj}\,A = \begin{bmatrix} 1 & -3 & 4 \\ -3 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$, Now $A^{-1} = \dfrac{\text{Adj}\,A}{|A|} = \dfrac{1}{(-4)} \begin{bmatrix} 1 & -3 & 4 \\ -3 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$

$$L.H.S = A \cdot A^{-1}$$

$$= \frac{1}{(-4)} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 4 \\ -3 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$$

$$= \frac{1}{(-4)} \begin{bmatrix} 1-6+1 & -3+2+1 & 4+0-4 \\ 3-6+3 & -9+2+3 & 12+0-12 \\ 1-3+2 & -3+1+2 & 4+0-8 \end{bmatrix}$$

$$= \frac{1}{(-4)} \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I = R.H.S$$

$$\therefore \boxed{AA^{-1} = I}$$

* If $A+B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $A-B = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ then find A^{-1} and B^{-1}.

→ $A+B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ — (1), $A-B = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ — (2)

Addition of (1) & (2) we have

$$\therefore A+\cancel{B}+A-\cancel{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

$$\therefore 2A = \begin{bmatrix} 4 & -2 \\ 4 & 6 \end{bmatrix} \Rightarrow A = \frac{1}{2} \begin{bmatrix} 4 & -2 \\ 4 & 6 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

→ Put the value of Matrix A in eqⁿ ①.

$$\therefore \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix} + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$

→ For A^{-1}

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix} = 6 + 2 = 8 \neq 0$$

$$\boxed{|A| = 8 \neq 0}$$

So, A^{-1} exists.

$$\therefore A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{8} \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

→ For B^{-1}

$$|B| = \begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} = -1 - 3 = -4 \neq 0$$

$$\boxed{|B| = -4 \neq 0}$$

So, B^{-1} exists.

$$\therefore B^{-1} = \frac{\text{Adj } B}{|B|} = \frac{1}{(-4)} \begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{4} \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$

* If $A+B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A-B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ then find $(AB)^{-1}$.

→ $A+B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ — (1) & $A-B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ — (2)

Addition of (1) & (2) we have.

$\therefore A+\cancel{B}+A-\cancel{B} = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$

$\therefore 2A = \begin{bmatrix} 4 & 0 \\ 4 & 4 \end{bmatrix} \Rightarrow A = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 4 & 4 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$

Now, put the value of matrix A in eqn (1).

$\therefore \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$

$\therefore B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$

$B = \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix}$.

Now, $AB = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} -2+0 & -2+0 \\ -2+2 & -2-4 \end{bmatrix}$

$= \begin{bmatrix} -2 & -2 \\ 0 & -6 \end{bmatrix}$

$\therefore |AB| = \begin{vmatrix} -2 & -2 \\ 0 & -6 \end{vmatrix} = 12 - 0 = 12 \neq 0$

$\therefore \boxed{|AB| = 12 \neq 0}$

So, $(AB)^{-1}$ exists.

$$\therefore (AB)^{-1} = \frac{Adj(AB)}{|AB|} = \frac{1}{12}\begin{bmatrix} -6 & 2 \\ 0 & -2 \end{bmatrix}$$

* Solve $2x + 3y = 6xy$ and $x - y = xy$ using Matrices.

$2x + 3y = 6xy$ — (1)

$x - y = xy$ — (2).

Now, dividing eq^n (1) & (2) by xy. so we have.

$$\therefore \frac{2\cancel{x}}{\cancel{x}y} + \frac{3\cancel{y}}{x\cancel{y}} = \frac{6\cancel{xy}}{\cancel{xy}}$$

$$\therefore \frac{2}{y} + \frac{3}{x} = 6.$$

And $x - y = xy$

$$\therefore \frac{\cancel{x}}{\cancel{x}y} - \frac{\cancel{y}}{x\cancel{y}} = \frac{\cancel{xy}}{\cancel{xy}}$$

$$\therefore \frac{1}{y} - \frac{1}{x} = 1.$$

Now, consider $\frac{1}{y} = a$ & $\frac{1}{x} = b$., so form of eq^n in new variable

$\therefore 2a + 3b = 6$

$a - b = 1.$

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

$\therefore AX = B$, where $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \end{bmatrix}$ and

$B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

$\therefore X = A^{-1} \cdot B$. — (3)

For that

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2-3 = -5 \neq 0$$

$$\therefore \boxed{|A| = -5 \neq 0}$$

So, A^{-1} exists.

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{(-5)} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix}$$

Now, From (3).

$$X = A^{-1} \cdot B$$

$$\therefore \begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{(-5)} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

$$= \frac{1}{(-5)} \begin{bmatrix} -6-3 \\ -6+2 \end{bmatrix}$$

$$= \frac{1}{(-5)} \begin{bmatrix} -9 \\ -4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 9/5 \\ 4/5 \end{bmatrix}$$

$\therefore a = 9/5$ & $b = 4/5$

But $a = \frac{1}{y} = \frac{9}{5}$ and $b = \frac{1}{x} = \frac{4}{5}$

$\therefore \boxed{x = \frac{5}{4}}$ & $\boxed{y = \frac{5}{9}}$

* Solve $x + y + z = 1$

$x + 2y + 3z = 4$

$x + 3y + 4z = 6$ using Matrices.

$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$

$AX = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ &

$B = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$

$\therefore X = A^{-1} \cdot B$. — (1).

For that, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{vmatrix}$

$= 1(8 - 9) - 1(4 - 3) + 1(3 - 2)$

$= 1(-1) - 1(1) + 1(1)$

$= -1 \cancel{-1} \cancel{+1}$

$= -1 \neq 0$

$\therefore \boxed{|A| = -1 \neq 0}$

$\therefore$ So, A^{-1} exists.

First we have to find co-factor of every element of matrix.

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$\rightarrow$ co-factor of $1 = (-1)^{1+1}\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8-9 = -1$

$\rightarrow$ co-factor of $1 = (-1)^{1+2}\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = -(4-3) = -1$

$\rightarrow$ co-factor of $1 = (-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3-2 = 1$

$\rightarrow$ co-factor of $1 = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = -(4-3) = -1$

$\rightarrow$ co-factor of $2 = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = 4-1 = 3$

$\rightarrow$ co-factor of $3 = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2.$

$\rightarrow$ co-factor of $1 = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3-2 = 1$

$\rightarrow$ co-factor of $3 = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$

$\rightarrow$ co-factor of $4 = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = 1$

$$\text{Adj } A = \begin{bmatrix} -1 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

Then, $A^{-1} = \dfrac{\text{Adj } A}{|A|} = \dfrac{1}{(-1)}\begin{bmatrix} -1 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

Now, from ①. $X = A^{-1}.B.$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)}\begin{bmatrix} -1 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = (-1) \begin{bmatrix} -1-4+6 \\ -1+12-12 \\ 1-8+6 \end{bmatrix}$$

$$= (-1) \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore \boxed{x = -1, \quad y = 1, \quad z = 1}$$

* Solve $2x - y + z = 1$
$x + 2y - z = 4$
$x - y + 2z = 1$ using Matrices.

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

$AX = B$, where $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$

$\therefore X = A^{-1}B$. — (1).

For that

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= 2(4-1) - (-1)(2+1) + 1(-1-2)$$

$$= 2(3) + 1(3) + 1(-3)$$

$$= 6 + 3 - 3$$

$$\therefore \boxed{|A| = 6 \neq 0}$$

so, A^{-1} exists.

Now, we have to find Adjoint of matrix A.

→ co-factor of 2 = $(-1)^{1+1}\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 4-1 = 3.$

→ co-factor of (-1) = $(-1)^{1+2}\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = -(2+1) = -3$

→ co-factor of 1 = $(-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = -1-2 = -3$

→ co-factor of 1 = $(-1)^{2+1}\begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} = -(-2+1) = 1$

→ co-factor of 2 = $(-1)^{2+2}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-1 = 3$

→ co-factor of (-1) = $(-1)^{2+3}\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-2+1) = 1$

→ co-factor of 1 = $(-1)^{3+1}\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} = 1-2 = -1$

→ co-factor of (-1) = $(-1)^{3+2}\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -(-2-1) = 3$

→ co-factor of 2 = $(-1)^{3+3}\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4+1 = 5$

$$\text{Adj}A = \begin{bmatrix} 3 & 1 & -1 \\ -3 & 3 & 3 \\ -3 & 1 & 5 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{\text{Adj}A}{|A|} = \dfrac{1}{6}\begin{bmatrix} 3 & 1 & -1 \\ -3 & 3 & 3 \\ -3 & 1 & 5 \end{bmatrix}$

Now, from (1) $X = A^{-1} \cdot B$.

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3 & 1 & -1 \\ -3 & 3 & 3 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 3+4-1 \\ -3+12+3 \\ -3+4+5 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6 \\ 12 \\ 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore \boxed{x = 1, \quad y = 2, \quad z = 1}$$

*. Solve $x - 2y + z = 1$
$2x - z = 3$
$x + y + 2z = 4$ using Matrices.

$x - 2y + z = 1$

$2x + 0y - z = 3$

$x + y + 2z = 4$

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$\therefore AX = B$, where $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $B = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$\therefore X = A^{-1}B$ — (1)

For that

$$|A| = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{vmatrix}$$

$= 1(0+1) + 2(4+1) + 1(2-0)$

$= 1(1) + 2(5) + 1(2)$

$= 1 + 10 + 2$

$= 13 \neq 0.$

Now, we have to find Adjoint of matrix.

For that we find co-factor of each element.

$\rightarrow$ Co-factor of $1 = (-1)^{1+1} \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 0 + 1 = 1$

$\rightarrow$ co-factor of $(-2) = (-1)^{1+2} \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = -(4+1) = -5$

$\rightarrow$ co-factor of $1 = (-1)^{1+3} \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2 - 0 = 2$

$\rightarrow$ co-factor of $2 = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} = -(-4-1) = 5$

$\rightarrow$ Co-factor of $0 = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1$

$\rightarrow$ co-factor of $(-1) = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} = -(1+2) = -3$

$\rightarrow$ co-factor of $1 = (-1)^{3+1}\begin{vmatrix} -2 & 1 \\ 0 & -1 \end{vmatrix} = 2-0 = 2$

$\rightarrow$ co-factor of $1 = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -(-1-2) = 3$

$\rightarrow$ co-factor of $2 = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ 2 & 0 \end{vmatrix} = 0+4 = 4.$

$$\text{Adj } A = \begin{bmatrix} 1 & 5 & 2 \\ -5 & 1 & 3 \\ 2 & -3 & 4 \end{bmatrix}$$

Now, $A^{-1} = \frac{\text{Adj } A}{|A|}$

$$\therefore A^{-1} = \frac{1}{13}\begin{bmatrix} 1 & 5 & 2 \\ -5 & 1 & 3 \\ 2 & -3 & 4 \end{bmatrix}$$

Now, From (1)

$$X = A^{-1} \cdot B$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 1 & 5 & 2 \\ -5 & 1 & 3 \\ 2 & -3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} 1+15+8 \\ -5+3+12 \\ 2-9+16 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 24 \\ 10 \\ 9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24/13 \\ 10/13 \\ 9/13 \end{bmatrix} \Rightarrow \boxed{x = \frac{24}{13},\ y = \frac{10}{13},\ z = \frac{9}{13}}$$

* solve $\frac{2}{x} + \frac{3}{y} - \frac{1}{z} = 10$

$\frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 5$

$\frac{2}{x} + \frac{2}{y} - \frac{3}{z} = 7$. using Matrices.

→ First consider $\frac{1}{x} = a$, $\frac{1}{y} = b$, $\frac{1}{z} = c$.

$2a + 3b - c = 10$

$a + 2b - c = 5$

$2a + 2b - 3c = 7$.

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & -1 \\ 2 & 2 & -3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 7 \end{bmatrix}$$

$A \quad X = B$, where $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & -1 \\ 2 & 2 & -3 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$B = \begin{bmatrix} 10 \\ 5 \\ 7 \end{bmatrix}$

$\therefore X = A^{-1} \cdot B$. —— (1).

For that

$$|A| = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 2 & -1 \\ 2 & 2 & -3 \end{vmatrix}$$

$= 2(-6+2) - 3(-3+2) - 1(2-4)$

$= 2(-4) - 3(-1) - 1(-2)$

$= -8 + 3 + 2$

$= -3 \neq 0$

$\therefore \boxed{|A| = -3 \neq 0}$

→ Now, find co-factor of each element of the matrix.

→ co-factor of 2 = $(-1)^{1+1}\begin{vmatrix} 2 & -1 \\ 2 & -3 \end{vmatrix} = -6+2 = -4$

→ co-factor of 3 = $(-1)^{1+2}\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} = -(-3+2) = 1$

→ co-factor of (-1) = $(-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 2-4 = -2$

→ co-factor of 1 = $(-1)^{2+1}\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix} = -(-9+2) = 7$

→ co-factor of 2 = $(-1)^{2+2}\begin{vmatrix} 2 & -1 \\ 2 & -3 \end{vmatrix} = -6+2 = -4$

→ co-factor of (-1) = $(-1)^{2+3}\begin{vmatrix} 2 & 3 \\ 2 & 2 \end{vmatrix} = -(4-6) = 2$

→ co-factor of 2 = $(-1)^{3+1}\begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} = -3+2 = -1$

→ co-factor of 2 = $(-1)^{3+2}\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-2+1) = 1$

→ co-factor of (-3) = $(-1)^{3+3}\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4-3 = 1.$

$$\text{Adj } A = \begin{bmatrix} -4 & 7 & -1 \\ 1 & -4 & 1 \\ -2 & 2 & 1 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{\text{Adj } A}{|A|}$

$$\therefore A^{-1} = \frac{1}{(-3)}\begin{bmatrix} -4 & 7 & -1 \\ 1 & -4 & 1 \\ -2 & 2 & 1 \end{bmatrix}$$

From ① we have

$$X = A^{-1} B.$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{(-3)} \begin{bmatrix} -4 & 7 & -1 \\ 1 & -4 & 1 \\ -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 7 \end{bmatrix}$$

$$= \frac{1}{(-3)} \begin{bmatrix} -40 + 35 - 7 \\ 10 - 20 + 7 \\ -20 + 10 + 7 \end{bmatrix}$$

$$= \frac{1}{(-3)} \begin{bmatrix} -12 \\ -3 \\ -3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore a = 4, \quad b = 1, \quad c = 1$

But $a = \frac{1}{x} = 4, \quad b = \frac{1}{y} = 1, \quad c = \frac{1}{z} = 1$

$$\therefore \boxed{x = \frac{1}{4}, \quad y = 1, \quad z = 1}$$

* Solve $\frac{3}{x} + \frac{6}{y} - \frac{3}{z} = 5,$

$\frac{2}{x} + \frac{2}{y} + \frac{3}{z} = 4,$

$\frac{1}{x} - \frac{4}{y} + \frac{6}{z} = 1,$ using Matrices.

→ Take $\frac{1}{x} = a, \; \frac{1}{y} = b, \; \frac{1}{z} = c.$

$3a+6b-3c=5$

$2a+2b+3c=4$

$a-4b+6c=1$

$$\begin{bmatrix}3 & 6 & -3\\2 & 2 & 3\\1 & -4 & 6\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}5\\4\\1\end{bmatrix}$$

$A\ X=B$, where $A=\begin{bmatrix}3 & 6 & -3\\2 & 2 & 3\\1 & -4 & 6\end{bmatrix}$, $X=\begin{bmatrix}a\\b\\c\end{bmatrix}$

and $B=\begin{bmatrix}5\\4\\1\end{bmatrix}$

$\therefore X=A^{-1}\cdot B$ —— ①.

For that

$$|A|=\begin{vmatrix}3 & 6 & -3\\2 & 2 & 3\\1 & -4 & 6\end{vmatrix}$$

$=3(12+12)-6(12-3)-3(-8-2)$

$=3(24)-6(9)-3(-10)$

$=72-54+30$

$=102-54$

$\therefore \boxed{|A|=48\neq 0}$

we have to find co-factor of each element of matrix A.

$\rightarrow$ co-factor of $3=(-1)^{1+1}\begin{vmatrix}2 & 3\\-4 & 6\end{vmatrix}=12+12=24$

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→ co-factor of $6 = (-1)^{1+2} \begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix} = -(12-3) = -9$

→ co-factor of $(-3) = (-1)^{1+3} \begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} = -8-2 = -10$

→ co-factor of $2 = (-1)^{2+1} \begin{vmatrix} 6 & -3 \\ -4 & 6 \end{vmatrix} = -(36-12) = -24$

→ co-factor of $2 = (-1)^{2+2} \begin{vmatrix} 3 & -3 \\ 1 & 6 \end{vmatrix} = 18+3 = 21$

→ co-factor of $3 = (-1)^{2+3} \begin{vmatrix} 3 & 6 \\ 1 & -4 \end{vmatrix} = -(-12-6) = 18$

→ co-factor of $1 = (-1)^{3+1} \begin{vmatrix} 6 & -3 \\ 2 & 3 \end{vmatrix} = 18+6 = 24$

→ co-factor of $(-4) = (-1)^{3+2} \begin{vmatrix} 3 & -3 \\ 2 & 3 \end{vmatrix} = -(9+6) = -15$

→ co-factor of $6 = (-1)^{3+3} \begin{vmatrix} 3 & 6 \\ 2 & 2 \end{vmatrix} = 6-12 = -6.$

$\therefore \text{Adj } A = \begin{bmatrix} 24 & -24 & 24 \\ -9 & 21 & -15 \\ -10 & 18 & -6 \end{bmatrix}$

Now, $A^{-1} = \dfrac{\text{Adj } A}{|A|}$ $\therefore A^{-1} = \dfrac{1}{48} \begin{bmatrix} 24 & -24 & 24 \\ -9 & 21 & -15 \\ -10 & 18 & -6 \end{bmatrix}$

Now, from (1).

$$X = A^{-1} \cdot B$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{48} \begin{bmatrix} 24 & -24 & 24 \\ -9 & 21 & -15 \\ -10 & 18 & -6 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} 120 - 96 + 24 \\ -45 + 84 - 15 \\ -50 + 72 - 6 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} 48 \\ 24 \\ 16 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ 1/3 \end{bmatrix}$$

$\therefore a = 1, \quad b = 1/2, \quad c = 1/3$

But $a = \frac{1}{x} = 1, \quad b = \frac{1}{y} = \frac{1}{2}, \quad c = \frac{1}{z} = \frac{1}{3}$

$$\therefore \boxed{x = 1, \quad y = 2, \quad z = 3}$$