UNIT-III

MARKS-18

TRIGONOMERY

(MCQS-04MARKS,EXAMPLES-14 MARKS)

UNIT-III - TRIGONOMETRY (18 MARKS IN GTU)

* convert Radian into degree form

  multiply by $\frac{\pi}{180}$.

* convert Degree into Radian form

  multiply by $\frac{180}{\pi}$.

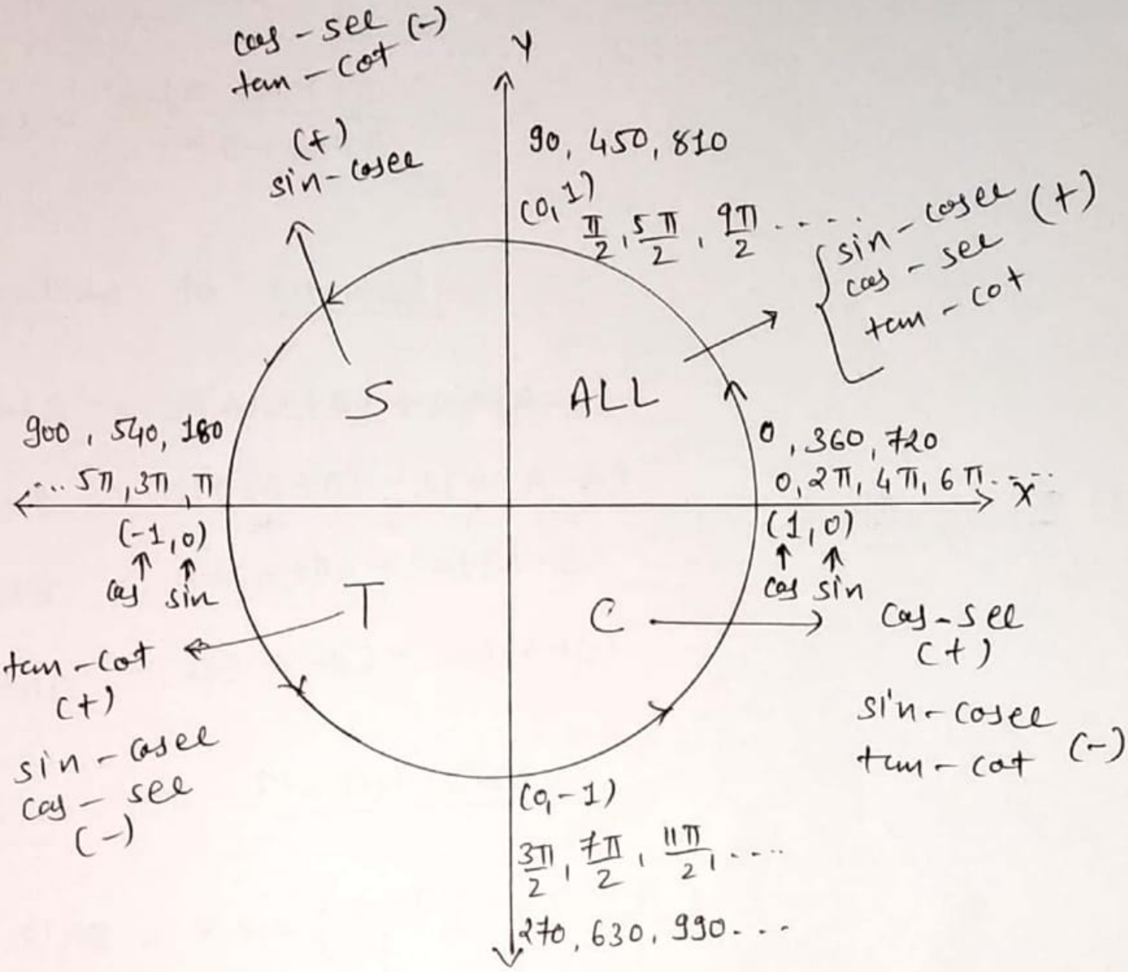
* <u>Values of Trigonometric Function</u>

Trigonometric Function / Angle	$0(0^\circ)$	$\frac{\pi}{6}(30^\circ)$	$\frac{\pi}{4}(45^\circ)$	$\frac{\pi}{3}(60^\circ)$	$\frac{\pi}{2}(90^\circ)$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0

* $\sin^2\theta + \cos^2\theta = 1$

  $\sec^2\theta - \tan^2\theta = 1$

  $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.



→ Function change at angle $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

In short, Function change on Y-axis's angles.

* $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

→ $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

· $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$.

→ Multiplication to summation

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

→ Summation to Multiplication

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

→ Multiple - submultiple Angle

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\tan 3\theta = \frac{3\tan\theta - (\tan\theta)^3}{1 - 3\tan^2\theta}$$

→ <u>Inverse</u> <u>Trigonometric</u> <u>Function</u> :-

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), \text{ if } xy < 1$$

$$= \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), \text{ if } xy > 1$$

$$= \frac{\pi}{2}, \text{ if } xy = 1$$

$$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}, \quad 0 < x < 1$$

$$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x}, \quad 0 < x < 1$$

$$\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}}, \quad x > 0$$

* Question for 1 Marks

1. Convert into degree.

$\frac{3\pi}{4}$, $\frac{7\pi}{6}$.

$\frac{3\cancel{\pi}}{\cancel{4}_{2}} \times \frac{\cancel{180}^{\cancel{90}\ 45}}{\cancel{\pi}} = 135^\circ$, $\frac{7\cancel{\pi}}{\cancel{6}} \times \frac{\cancel{180}^{30}}{\cancel{\pi}} = 210^\circ$

2. Convert into Radian. 150°, 20°

$150^\circ = \cancel{150}^{5} \times \frac{\pi}{\cancel{180}_{6}} = \frac{5\pi}{6}$.

$20^\circ = \cancel{20} \times \frac{\pi}{\cancel{180}_{9}} = \frac{\pi}{9}$

3. If $\cos\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = -\frac{1}{2}$ then θ lies in 4^{th} Quadrant because cos is positive and sin is negative.

4. $\cos 90 \times \cos 60 \times \sin 30$.

$= 0 \times \frac{1}{2} \times \frac{1}{2}$

$= \boxed{0}$

5. $\tan 225^\circ$

$= \tan(180+45) = \tan 45 = \boxed{1}$

6. $\cot(-30^\circ)$

$= -\cot 30^\circ = \boxed{-\sqrt{3}}$

7. $\sin^2 57 + \sin^2 33$

$= \sin^2 57 + \sin^2(90-57)$

$= \sin^2 57 + \cos^2 57.$

$= \boxed{1}$

8. $\cos^2 30^\circ + \cos^2 60^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = \boxed{1}$

9. If $A+B+C = \pi$ then $\cos\left(\frac{B+C}{2}\right)$ is

$B+C = \pi - A$

$\therefore \frac{B+C}{2} = \frac{\pi - A}{2}.$

Now, $\cos\left(\frac{B+C}{2}\right) = \cos\left(\frac{\pi - A}{2}\right)$

$= \cos\left(\frac{\pi}{2} - \frac{A}{2}\right)$

$= \sin A/2.$

10. $\cos x + \cos(\pi - x) + \cos(2\pi - x) + \cos(3\pi - x)$

$= \cos x + (-\cos x) + \cos x + (-\cos x)$

$= \cancel{\cos x} - \cancel{\cos x} + \cancel{\cos x} - \cancel{\cos x}$

$= 0$

11. period of $\sin 2x = \frac{2\pi}{2} = \pi$.

12. period of $\tan\left(3x + \frac{\pi}{6}\right) = \frac{\pi}{3}$

13. $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

14. $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

15. $\cos\left(\cos^{-1}\frac{2}{3}\right) = 2/3$

16. $\sin(\tan^{-1} p + \cot^{-1} p)$

$= \sin\frac{\pi}{2} = \boxed{1}$

* <u>Question for 3 Marks</u>:-

1. Prove that $\tan 225^\circ \times \cot 405^\circ + \tan 765^\circ \times \cot 675^\circ = 0$.

L.H.S $= \tan 225^\circ \times \cot 405 + \tan 765 \times \cot 675$

$= \tan(180+45) \times \cot(360+45) + \tan(720+45^\circ) + \cot(630+45$

$= \tan 45 \times \cot 45 + \tan 45 * (-\tan 45)$

$= 1 - (1)(1)$

$= 1 - 1$

$= 0$

$=$ R.H.S

2. Prove that $\cos\frac{19\pi}{6}\sin\frac{17\pi}{6} - \sin\frac{11\pi}{6}\cos\frac{13\pi}{6} = 0.$

L.H.S :- $\cos\frac{19\pi}{6}\sin\frac{17\pi}{6} - \sin\frac{11\pi}{6}\cos\frac{13\pi}{6}$

$= \cos\left(3\pi + \frac{\pi}{6}\right)\sin\left(3\pi - \frac{\pi}{6}\right) - \sin\left(2\pi - \frac{\pi}{6}\right)\cos\left(2\pi + \frac{\pi}{6}\right)$

$= - \cos\frac{\pi}{6}\sin\frac{\pi}{6} - \left(-\sin\frac{\pi}{6}\right)\cos\frac{\pi}{6}$

$= - \cos\frac{\pi}{6}\sin\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\pi/6$

$= - \frac{\sqrt{3}}{2}\frac{1}{2} + \frac{1}{2}\frac{\sqrt{3}}{2}$

$= \frac{-\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= 0$

$=$ R.H.S

3. prove that $\sin^2\frac{\pi}{4} + \sin^2\frac{3\pi}{4} + \sin^2\frac{5\pi}{4} + \sin^2\frac{7\pi}{4} = 2.$

L.H.S $= \sin^2\frac{\pi}{4} + \sin^2\frac{3\pi}{4} + \sin^2\frac{5\pi}{4} + \sin^2\frac{7\pi}{4}$

$= \sin^2\frac{\pi}{4} + \sin^2\left(\pi - \frac{\pi}{4}\right) + \sin^2\left(\pi + \frac{\pi}{4}\right) + \sin^2\left(2\pi - \frac{\pi}{4}\right)$

$= \sin^2\frac{\pi}{4} + \sin^2\frac{\pi}{4} + \sin^2\frac{\pi}{4} + \sin^2\frac{\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2 =$ R.H.S.

4. Prove that $\tan\frac{\pi}{20}\tan\frac{3\pi}{20}\tan\frac{5\pi}{20}\tan\frac{7\pi}{20}\tan\frac{9\pi}{20} = 1$

$\rightarrow$ $\tan\frac{9\pi}{20} = \tan\left(\frac{\pi}{2} - \frac{\pi}{20}\right) = \cot\frac{\pi}{20}$

$\tan\frac{7\pi}{20} = \tan\left(\frac{\pi}{2} - \frac{3\pi}{20}\right) = \cot\frac{3\pi}{20}$

$\tan\frac{5\pi}{20} = \tan\frac{\pi}{4} = 1$

$L.H.S = \tan\frac{\pi}{20}\tan\frac{3\pi}{20}\tan\frac{5\pi}{20}\tan\frac{7\pi}{20}\tan\frac{9\pi}{20}$

$= \tan\frac{\pi}{20}\cdot\tan\frac{3\pi}{20}\ 1\cdot\cos\frac{3\pi}{20}\cdot\cot\frac{\pi}{20}$

$= \left(\tan\frac{\pi}{20}\cot\frac{\pi}{20}\right)\cdot\left(\tan\frac{3\pi}{20}\cot\frac{3\pi}{20}\right)$

$= (1)\ (1)$

$= 1$

$= R.H.S.$

5. P. T. $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B$

$= \cos^2 B - \cos^2 A.$

$L.H.S = \sin(A+B)\cdot\sin(A-B)$

$= (\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$

$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\cdot\sin^2 B.$

$(\sin^2\theta + \cos^2\theta = 1)$

$= \sin^2 A - \cancel{\sin^2 A\sin^2 B} - \sin^2 B + \cancel{\sin^2 A\sin^2 B}$

$= \sin^2 A - \sin^2 B = R.H.S$

$= \sin^2 A - \sin^2 B$

$= 1 - \cos^2 A - (1 - \cos^2 B)$

$= \cancel{1} - \cos^2 A - \cancel{1} + \cos^2 B$

$= \cos^2 B - \cos^2 A$

$=$ R.H.S

6. Prove that $\cos A \cdot \sin(B-C) + \cos B \sin(C-A) + \cos C \sin(A-B) = 0$

$\cos A \cdot \sin(B-C)$

$= \cos A (\sin B \cos C - \cos B \sin C)$

$= \cos A \sin B \cos C - \cos A \cos B \sin C$

Similarly, $\cos B \sin(C-A)$

$= \cos B \sin C \cos A - \cos B \sin A \cos C$

$= \cos A \cos B \sin C - \sin A \cos B \cos C$

$\rightarrow \cos C \sin(A-B)$

$\sin A \cos B \cos C - \cos A \sin B \cos C$

L.H.S $= \cos A \cdot \sin(B-C) + \cos B \sin(C-A) + \cos C \sin(A-B)$

$= \cancel{\cos A \sin B \cos C} - \cancel{\cos A \cos B \sin C}$
$+ \cancel{\cos A \cos B \sin C} - \cancel{\sin A \cos B \cos C}$
$+ \cancel{\sin A \cos B \cos C} - \cancel{\cos A \sin B \cos C}$

$= 0$

$=$ R.H.S

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7. prove that $\tan 55^\circ = \dfrac{\cos 10 + \sin 10}{\cos 10 - \sin 10}$

$$L.H.S = \tan 55$$

$$= \tan(45 + 10^\circ)$$

$$= \frac{\tan 45 + \tan 10}{1 - \tan 45 \tan 10}$$

$$= \frac{1 + \tan 10}{1 - \tan 10} \quad (\because \tan 45 = 1)$$

$$= \frac{1 + \frac{\sin 10}{\cos 10}}{1 - \frac{\sin 10}{\cos 10}}$$

$$= \frac{\cos 10 + \sin 10}{\cos 10 - \sin 10} = R.H.S.$$

8. prove that $\dfrac{\sin 79 + \sin 19}{\cos 19 - \cos 79} = \sqrt{3}$

$$L.H.S = \frac{\sin 79 + \sin 19}{\cos 19 - \cos 79}$$

$$= \frac{2\sin\left(\frac{79+19}{2}\right)\cos\left(\frac{79-19}{2}\right)}{-2\sin\left(\frac{19+79}{2}\right)\sin\left(\frac{19-79}{2}\right)}$$

$s + s = 2sc$

$c - c = -2ss$

$$= \frac{2\sin\left(\frac{98}{2}\right)\cos\left(\frac{60}{2}\right)}{-2\sin\left(\frac{98}{2}\right)\sin\left(\frac{-60}{2}\right)}$$

$$= \frac{\cancel{2\sin 49^\circ}\cos 30^\circ}{-\cancel{2\sin 49^\circ}\sin(-30^\circ)}$$

$$= \frac{\cos 30^\circ}{\sin 30^\circ} \qquad (\because \sin(-\theta) = -\sin\theta)$$

$$= \cot 30^\circ$$

$$= \sqrt{3}$$

$$= R.H.S.$$

$\therefore L.H.S = R.H.S$

9. P. T. $(1 + \tan 25^\circ)(1 + \tan 20^\circ) = 2.$

$\rightarrow \tan 45^\circ = \tan(25^\circ + 20^\circ)$

$$\therefore 1 = \frac{\tan 25 + \tan 20}{1 - \tan 25 \tan 20}$$

$\therefore 1 - \tan 25 \tan 20^\circ = \tan 25^\circ + \tan 20^\circ$

$\therefore \tan 25^\circ + \tan 20^\circ + \tan 25^\circ \tan 20^\circ = 1.$

Adding 1 on both side:

$\therefore 1 + \tan 25^\circ + \tan 20^\circ + \tan 25^\circ \tan 20^\circ = 1 + 1$

$\therefore 1(1 + \tan 25^\circ) + \tan 20^\circ (1 + \tan 25^\circ) = 2.$

$\therefore (1 + \tan 25^\circ)(1 + \tan 20^\circ) = 2$

10. If $\tan x = \frac{5}{6}$ and $\tan y = \frac{1}{11}$ then P. T.

$x + y = \frac{\pi}{4}$

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$\rightarrow \quad \tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$.

$$= \frac{5/6 + 1/11}{1 - 5/6 \cdot 1/11}$$

$$= \frac{\frac{55+6}{66}}{\frac{66-5}{66}}$$

$$= \frac{61/66}{61/66}$$

$\therefore \tan(x+y) = 1 > 0 \quad —— \quad (1)$

$\therefore \quad 0 < x < \frac{\pi}{2} \quad \& \quad 0 < y < \frac{\pi}{2}$

$\therefore 0 < x+y < \pi$

$\therefore x + y = \tan^{-1} 1$

$\therefore \boxed{x + y = \frac{\pi}{4}}$

11. P. T. $\sin 3A = 3\sin A - 4\sin^3 A$

L.H.S $= \sin 3A$

$= \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cos A \cos A + (1 - 2\sin^2 A) \sin A$

$= 2\sin A \cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A (1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

$=$ R.H.S.

12. P.T. $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$

L.H.S $= \cos 20° \cos 40° \cos 80°$

$= \frac{1}{2} \cos 20° [2 \cos 80 \cdot \cos 40]$

$(\because 2CC = C + C)$

$= \frac{1}{2} \cos 20° [\cos(80+40) + \cos(80-40)]$

$= \frac{1}{2} \cos 20° [\cos 120 + \cos 40]$

$= \frac{1}{2} \cos 20° [\cos(180-60) + \cos 40]$

$= \frac{1}{2} \cos 20° [-\cos 60 + \cos 40]$

$= \frac{1}{2} \cos 20 \left(-\frac{1}{2}\right) + \frac{1}{2} \cos 40 \cos 20°$

$= -\frac{1}{4} \cos 20 + \frac{1}{2} \frac{1}{2} [2 \cos 40 \cos 20°]$

$= -\frac{1}{4} \cos 20 + \frac{1}{4} [\cos(40+20°) + \cos(40-20°)]$

$= -\frac{1}{4} \cos 20 + \frac{1}{4} [\cos 60 + \cos 20]$

$= -\frac{1}{4} \cos 20 + \frac{1}{4} \left(\frac{1}{2} + \cos 20°\right)$

$= -\frac{1}{4} \cancel{\cos 20} + \frac{1}{8} + \frac{1}{4} \cancel{\cos 20°}$

$= \frac{1}{8}$

$=$ R.H.S

13. Prove that $\frac{\sin 19 + \cos 11}{\cos 19 - \sin 11} = \sqrt{3}$

L.H.S. $= \frac{\sin 19 + \cos 11}{\cos 19 - \sin 11}$

$= \frac{\sin 19 + \cos(90-79)}{\cos 19 - \sin(90-79)}$

$= \frac{\sin 19 + \sin 79}{\cos 19 - \cos 79}$

$= \frac{2\sin\left(\frac{19+79}{2}\right)\cos\left(\frac{19-79}{2}\right)}{-2\sin\left(\frac{19+79}{2}\right)\sin\left(\frac{19-79}{2}\right)}$

$\left(\because \begin{array}{l} S+S = 2SC \\ C-C = -2SS \end{array}\right)$

$= \frac{\cancel{2\sin 49} \cos(-30^\circ)}{-\cancel{2\sin 49^\circ} \sin(-30^\circ)}$

$= \frac{\cos 30}{\sin 30}$ $\left(\because \begin{array}{l} \cos(-\theta) = \cos\theta \\ \sin(-\theta) = -\sin\theta \end{array}\right)$

$= \cot 30^\circ$

$= \sqrt{3}$

$=$ R.H.S.

14. prove that $\sin 4\theta = \frac{4\tan\theta\,(1-\tan^2\theta)}{(1+\tan^2\theta)^2}$

L.H.S $= \sin 4\theta$

$= 2\sin 2\theta \cos 2\theta$

$= 2\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

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$$= \frac{4\tan\theta\,(1-\tan^2\theta)}{(1+\tan^2\theta)^2}$$

$= R.H.S$

15. Prove that $\dfrac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A} = \tan A$

$$L.H.S = \frac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A}$$

$$= \frac{(1-\cos 2A)+\sin 2A}{(1+\cos 2A)+\sin 2A}$$

$$= \frac{2\sin^2 A + 2\sin A\cos A}{2\cos^2 A + 2\sin A\cos A}$$

$$= \frac{\cancel{2}\sin A\,(\sin A + \cos A)}{\cancel{2}\cos A\,(\cos A + \sin A)}$$

$= \tan A$

$= R.H.S.$

16. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

$$L.H.S = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$$

$\left(x \cdot y = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6} < 1\right)$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3+2}{\cancel{6}}}{\frac{6-1}{\cancel{6}}}\right)$$

$$= \tan^{-1}\left(\frac{5}{5}\right) = \tan^{-1}1 = \frac{\pi}{4} = R.H.S$$

* Question for 4 Marks :-

1. prove that $\dfrac{\sin(\theta-\frac{\pi}{2})}{\cos(\theta-\pi)} + \dfrac{\tan(\frac{\pi}{2}-\theta)}{\cot(\pi-\theta)} + \dfrac{\operatorname{cosec}(\frac{\pi}{2}+\theta)}{\sec(\pi+\theta)} = -1$

$$\text{L.H.S} = \frac{\sin(\theta-\frac{\pi}{2})}{\cos(\theta-\pi)} + \frac{\tan(\frac{\pi}{2}-\theta)}{\cot(\pi-\theta)} + \frac{\operatorname{cosec}(\frac{\pi}{2}+\theta)}{\sec(\pi+\theta)}$$

$$= \frac{\sin\left[-(\frac{\pi}{2}-\theta)\right]}{\cos\left[-(\pi-\theta)\right]} + \frac{\tan(\frac{\pi}{2}-\theta)}{\cot(\pi-\theta)} + \frac{\operatorname{cosec}(\frac{\pi}{2}+\theta)}{\sec(\pi+\theta)}$$

$$= \frac{-\sin(\frac{\pi}{2}-\theta)}{\cos(\pi-\theta)} + \frac{\tan(\frac{\pi}{2}-\theta)}{\cot(\pi-\theta)} + \frac{\operatorname{cosec}(\frac{\pi}{2}+\theta)}{\sec(\pi+\theta)}$$

$$= \frac{-\cos\theta}{-\cos\theta} + \frac{\cot\theta}{-\cot\theta} + \frac{\sec\theta}{-\operatorname{cosec}\theta}$$

$$= \cancel{1} - \cancel{1} - 1$$

$$= -1$$

$$= \text{R.H.S}$$

2. prove that $\dfrac{\sin\theta + \sin 2\theta + \sin 4\theta + \sin 5\theta}{\cos\theta + \cos 2\theta + \cos 4\theta + \cos 5\theta} = \tan 3\theta$

$$\text{L.H.S} = \frac{\sin\theta + \sin 2\theta + \sin 4\theta + \sin 5\theta}{\cos\theta + \cos 2\theta + \cos 4\theta + \cos 5\theta}$$

$$= \frac{\sin 5\theta + \sin\theta + \sin 4\theta + \sin 2\theta}{\cos 5\theta + \cos\theta + \cos 4\theta + \cos 2\theta}$$

$$= \frac{2\sin\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right) + 2\sin\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right)}{2\cos\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right) + 2\cos\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right)}$$

$$= \frac{2\sin 3\theta \cos 2\theta + 2\sin 3\theta \cos\theta}{2\cos 3\theta \cos 2\theta + 2\cos 3\theta \cos\theta}$$

$$= \frac{\cancel{2}\sin 3\theta (\cancel{\cos 2\theta + \cos\theta})}{\cancel{2}\cos 3\theta (\cancel{\cos 2\theta + \cos\theta})}$$

$= \tan 3\theta$

$=$ R. H. S

3.* P. T. $\sin(180^\circ - \theta)\cos(-\theta)\cot(180^\circ - \theta) + \cos(360^\circ + \theta)$
$\operatorname{cosec}(180 - \theta)\cot(90 - \theta) = \sin^2\theta.$

L.H.S $= \sin(180 - \theta)\cos(-\theta)\cot(180 - \theta) + \cos(360^\circ + \theta)$
$\operatorname{cosec}(180 - \theta)\cot(90 - \theta)$

$= \sin(\pi - \theta)\cos(-\theta)\cot(\pi - \theta) + \cos(2\pi + \theta)$
$\operatorname{cosec}(\pi - \theta)\cot(\frac{\pi}{2} - \theta)$

$= (\sin\theta)(\cos\theta)(-\cot\theta) + (\cos\theta)(\operatorname{cosec}\theta)(\tan\theta)$

$= \cancel{\sin\theta} \cdot \cos\theta \left(-\frac{\cos\theta}{\cancel{\sin\theta}}\right) + \cancel{\cos\theta}\left(\frac{1}{\cancel{\sin\theta}}\right)\left(\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}}\right)$

$= -\cos^2\theta + 1$

$= 1 - \cos^2\theta$

$= \sin^2\theta$

$=$ R. H. S.

4. P. T. $\dfrac{\tan(\pi - \theta)}{\tan(\pi + \theta)} \times \dfrac{\cot(\pi + \theta)}{\cot(\pi - \theta)} \times \dfrac{\tan(2\pi + \theta)}{\cot(2\pi - \theta)} = -\tan^2\theta$

$$L.H.S = \frac{\tan(\pi-\theta)}{\tan(\pi+\theta)} \times \frac{\cot(\pi+\theta)}{\cot(\pi-\theta)} \times \frac{\tan(2\pi+\theta)}{\cot(2\pi-\theta)}$$

$$= \frac{-\cancel{\tan\theta}}{\cancel{\tan\theta}} \times \frac{\cancel{\cot\theta}}{-\cancel{\cot\theta}} \times \frac{\tan\theta}{-\cot\theta}$$

$$= (-1) \times (-1) \times (\tan\theta)(-\tan\theta)$$

$$= -\tan^2\theta$$

$$= R.H.S.$$

5. If A, B, C & D are angles of cyclic quadrilateral then P. T. $\cos A + \cos B + \cos C + \cos D = 0$.

→ $\square ABCD$ is a cyclic quadrilateral.

In cyclic quadrilateral sum of opposite angle is 180°

$\therefore m\angle A + m\angle C = 180°$ — (1)

$m\angle B + m\angle D = 180°$ — (2)

Now, From (1) $A + C = \pi$

$\therefore C = \pi - A$

$\therefore \cos C = \cos(\pi - A)$

$\therefore \cos C = -\cos A$ — (3)

From (2), $B + D = \pi$

$\therefore D = \pi - B$

$\therefore \cos D = \cos(\pi - B)$

$\therefore \cos D = -\cos B$ — (4)

$L.H.S = \cos A + \cos B + \cos C + \cos D$

$= \cancel{\cos A} + \cancel{\cos B} - \cancel{\cos A} - \cancel{\cos B}$ ($\because$ by (3) & (4))

$= 0 = R.H.S.$

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6. For any ΔABC P.T. $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

$\rightarrow$ we know that for any ΔABC.

$$A + B + C = \pi$$

$$\therefore A + B = \pi - C$$

$$\therefore \tan(A+B) = \tan(\pi - C)$$

$$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$$\therefore \tan A + \tan B = -\tan C \, (1 - \tan A \tan B)$$

$$\therefore \tan A + \tan B = -\tan C + \tan A \tan B \tan C$$

$$\therefore \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

7. Prove that $4 \sin 2A \sin(60^\circ + 2A) \sin(60^\circ - 2A) = \sin 6A$

$$\text{L.H.S} = 4 \sin 2A \sin(60 + 2A) \sin(60 - 2A)$$

$$= -2 \sin 2A \left[-2 \sin(60 + 2A) \sin(60 - 2A)\right]$$

$$= -2 \sin 2A \left[\cos(60 + 2A + 60 - 2A) - \cos(60 + 2A - 60 + 2A)\right]$$

$$= -2 \sin 2A \left[\cos 120^\circ - \cos 4A\right]$$

$$= -2 \sin 2A \left[\cos(180 - 60) - (\cos 4A)\right]$$

$$= -2 \sin 2A \left[-\cos 60 - \cos 4A\right]$$

$$= -2 \sin 2A \left[-\tfrac{1}{2} - \cos 4A\right]$$

$$= \sin 2A + 2 \cos 4A \sin 2A$$

$$= \cancel{\sin 2A} + \sin 6A - \cancel{\sin 2A} \qquad (\because 2CS = S - S)$$

$$= \sin 6A = \text{R.H.S}$$

8. If $\tan\theta = \frac{1}{2}$ then P.T. $7\cos2\theta + 8\sin2\theta = \frac{53}{5}$

L.H.S $= 7\cos2\theta + 8\sin2\theta$

$$= 7\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 8\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= 7\left(\frac{1-(\frac{1}{2})^2}{1+(\frac{1}{2})^2}\right) + 8\left(\frac{2\cdot\frac{1}{2}}{1+(\frac{1}{2})^2}\right)$$

$$= 7\left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right) + 8\left(\frac{1}{1+\frac{1}{4}}\right)$$

$$= 7\left(\frac{\frac{4-1}{4}}{\frac{4+1}{4}}\right) + 8\left(\frac{4}{4+1}\right)$$

$$= 7\left(\frac{3}{5}\right) + 8\left(\frac{4}{5}\right)$$

$$= \frac{21+32}{5}$$

$$= \frac{53}{5} = \text{R.H.S.}$$

9. P.T. $\cos20^\circ + \cos60 + \cos100 + \cos140 = \frac{1}{2}$

L.H.S $= \cos20 + \cos60 + \cos100 + \cos140.$

$$= \cos60 + (\cos140 + \cos100) + \cos20^\circ$$

$$= \frac{1}{2} + 2\cos\left(\frac{140+100}{2}\right)\cos\left(\frac{140-100}{2}\right) + \cos20^\circ$$

$$= \frac{1}{2} + 2\cos120\cos20 + \cos20.$$

$$= \frac{1}{2} + 2\cos(180-60)\cos20 + \cos20.$$

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$= \frac{1}{2} + 2(-\cos 60^\circ)\cos 20 + \cos 20^\circ$

$= \frac{1}{2} + \cancel{2}\cdot\left(-\frac{1}{\cancel{2}}\right)\cos 20 + \cos 20^\circ$

$= \frac{1}{2} - \cancel{\cos 20} + \cancel{\cos 20}$

$= 1/2 = R.H.S.$

10. Prove that $\sin 10^\circ \sin 30 \sin 50 \sin 70 = 1/16$

L.H.S $= \sin 10 \sin 30 \sin 50 \sin 70$

$= -\frac{1}{2} \sin 10^\circ \frac{1}{2}[-2\sin 70 \sin 50]$

$= -\frac{1}{4} \sin 10^\circ [\cos 120 - \cos 20]$ $(\because -2SS = C - C)$

$= -\frac{1}{4} \sin 10^\circ [\cos(180-60) - \cos 20^\circ]$

$= -\frac{1}{4} \sin 10^\circ [-\cos 60 - \cos 20^\circ]$

$= -\frac{1}{4} \sin 10^\circ [-\frac{1}{2} - \cos 20^\circ]$

$= \frac{1}{8} \sin 10^\circ + \frac{1}{4} \cos 20^\circ \sin 10^\circ$

$= \frac{1}{8} \sin 10^\circ + \frac{1}{4} \cdot \frac{1}{2} (2\cos 20^\circ \sin 10^\circ)$

$= \frac{1}{8} \sin 10 + \frac{1}{8} [\sin 30 - \sin 10]$ $(\because 2CS = S - S)$

$= \frac{1}{8} \sin 10 + \frac{1}{8} [\frac{1}{2} - \sin 10]$

$= \frac{1}{8} \cancel{\sin 10} + \frac{1}{16} - \frac{1}{8} \cancel{\sin 10}$

$= \frac{1}{16} = R.H.S.$

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11. Prove that $\tan^{-1}\left(\frac{3}{4}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$.

$$\text{L.H.S} = \tan^{-1}\left(\frac{3}{4}\right) + \sin^{-1}\left(\frac{4}{5}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{4/5}{\sqrt{1-16/25}}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{4/5}{\sqrt{9/25}}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{4/\cancel{5}}{3/\cancel{5}}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{4}{3}\right)$$

$$= \frac{\pi}{2} \qquad \left(\because x \cdot y = \frac{3}{4}\cdot\frac{4}{3} = 1\right)$$

$$= \text{R.H.S}$$

12. P.T. $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{13}\right) = \pi$

$$\text{L.H.S} = \sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{13}\right)$$

$$= \tan^{-1}\left(\frac{12/13}{\sqrt{1-\frac{144}{169}}}\right) + \tan^{-1}\left(\frac{\sqrt{1-16/25}}{4/5}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \tan^{-1}\left(\frac{12/13}{\sqrt{25/169}}\right) + \tan^{-1}\left(\frac{\sqrt{9/25}}{4/5}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \tan^{-1}\left(\frac{12/\cancel{13}}{5/\cancel{13}}\right) + \tan^{-1}\left(\frac{3/5}{4/5}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \tan^{-1}\left(\frac{12}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \pi + \tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\cdot\frac{3}{4}}\right) + \tan^{-1}\left(\frac{63}{16}\right) \qquad \left(\because x\cdot y = \frac{12}{5}\cdot\frac{3}{4} = \frac{36}{20} > 1\right)$$

$$= \pi + \tan^{-1}\left(\frac{\frac{48+15}{20}}{\frac{20-36}{20}}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \pi + \tan^{-1}\left(\frac{63/20}{-16/20}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \pi + \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\frac{63}{16}$$

$$= \pi - \cancel{\tan^{-1}\frac{63}{16}} + \cancel{\tan^{-1}\frac{63}{16}} \qquad (\because \tan^{-1}(-\theta) = -\tan^{-1}\theta)$$

$$= \pi$$

$= R.H.S.$

13. Draw the graph of $y = \sin x$, $0 \le x \le 2\pi$

14. Draw the graph of $y = \cos x$, $0 \le x \le 2\pi$

15. Draw the graph of $y = \sin 2x$, $-\pi \le x \le \pi$

16. Draw the graph of $y = 3\cos\left(\frac{x}{2}\right)$, $0 \le x \le 2\pi$