UNIT-IV

MARKS-14

VECTOR

(MCQS-00MARKS,EXAMPLES-14 MARKS)

UNIT- IV

VECTOR (14 Marks)

→ Vector is denoted by $\bar{x} = (x_1, x_2, x_3) = x_1 i + x_2 j + x_3 k$.

→ Modulus of vector is defined as

$$|\bar{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

→ If $\bar{x} = (x_1, x_2, x_3)$ and $\bar{y} = (y_1, y_2, y_3)$ then Addition & Substraction of that two Vector is

$$\bar{x} + \bar{y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3) \text{ and}$$

$$\bar{x} - \bar{y} = (x_1 - y_1, x_2 - y_2, x_3 - y_3).$$

→ Unit vector If $|\bar{x}| = 1$. then vector $\bar{x}$ is called Unit vector.

→ Direction cosines of Vector :-

$$l = \frac{x}{\sqrt{x^2+y^2+z^2}}, \quad m = \frac{y}{\sqrt{x^2+y^2+z^2}}, \quad n = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

→ Dot product of two Vectors

$$\bar{x} \cdot \bar{y} = (x_1, x_2, x_3) \cdot (y_1, y_2, y_3)$$

$$= x_1 y_1 + x_2 y_2 + x_3 y_3.$$

→ $\bar{x} \cdot \bar{y} = \bar{y} \cdot \bar{x}$

• Cross product of two vectors

$$\bar{x} \times \bar{y} = \begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \qquad \begin{aligned} i &= (1,0,0) \\ j &= (0,1,0) \\ k &= (0,0,1) \end{aligned}$$

→ $\bar{x} \times \bar{y} = -\bar{y} \times \bar{x}$

* Angle between two vector

$$\bar{a} \cdot \bar{b} = |\bar{a}|\,|\bar{b}| \cos\theta$$

and

$$|\bar{x} \times \bar{y}| = |\bar{x}|\,|\bar{y}| \sin\theta.$$

* Work-done by Force

$W = \bar{F} \cdot \bar{d}$, where $\bar{F} = \bar{F_1} + \bar{F_2}$ = Total Force

$\bar{d} = d_2 - d_1$ = displacement

To point − From point

* perpendicular Unit Vector

→ To find perpendicular Unit Vector.

$$\frac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|}$$

→ If $\bar{a}$ & $\bar{b}$ are perpendicular to each other then

$$\bar{a} \cdot \bar{b} = 0.$$

→ If $\bar{a}$

* Question for 1 Marks :-

1. If $\bar{a} = +2i + 3j + k$, $\bar{b} = 2i - 3j + 2k$ then $\bar{a} + \bar{b} =$ ______

Here $\bar{a} = (2, 3, 1)$, $\bar{b} = (2, -3, 2)$

$$\bar{a} + \bar{b} = (2, 3, 1) + (2, -3, 2)$$

$$= (4, 0, 3)$$

$$\therefore \boxed{\bar{a} + \bar{b} = 4i + 3k}$$

2>. If $\bar{a} = 2i+3j$, $\bar{b} = 3i-j-2k$ then $\bar{a}-\bar{b}$ is.

Here $\bar{a} = (2, 3, 0)$

$\bar{b} = (3, -1, -2)$

$\bar{a}-\bar{b} = (2, 3, 0) - (3, -1, -2)$

$= (2-3, 3+1, 0+2)$

$= (-1, 4, 2)$

$\therefore \bar{a}-\bar{b} = -i+4j+2k.$

3>. If $\bar{a} = -i+3j$ then $|\bar{a}| =$ ____

Here $\bar{a} = -i+3j$

$\bar{a} = (-1, 3)$

$|\bar{a}| = \sqrt{(-1)^2+3^2} = \sqrt{1+9} = \sqrt{10}$

$\therefore \boxed{|\bar{a}| = \sqrt{10}}$

4> If $\bar{u} = \frac{1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j$ then $|\bar{u}| =$ ____

Here $\bar{u} = \frac{1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j$

$\bar{u} = (1/\sqrt{5}, 2/\sqrt{5})$

$\therefore |\bar{u}| = \sqrt{(1/\sqrt{5})^2+\left(\frac{2}{\sqrt{5}}\right)^2} = \sqrt{1/5+4/5} = \sqrt{\frac{1+4}{5}} = \sqrt{5/5} = \sqrt{1} = 1.$

$\therefore \boxed{|\bar{u}| = 1}$

5> If $\bar{a} = 3i-4j-5\sqrt{3}k$ then $|\bar{a}| =$ ____

Here $\bar{a} = (3, -4, -5\sqrt{3})$

so, $|\bar{a}| = \sqrt{(3)^2+(-4)^2+(-5\sqrt{3})^2}$

$= \sqrt{9+16+75}$

$= \sqrt{100}$

$\therefore \boxed{|\bar{a}| = 10}$

6> If $\bar{a} = -i+3j$ and $\bar{b} = 2i+3j$, then $|\bar{a}| + |\bar{b}| = \underline{\sqrt{10}+\sqrt{13}}$

Here $\bar{a} = (-1, 3)$ & $\bar{b} = (2, 3)$

$|\bar{a}| = \sqrt{(-1)^2+3^2} = \sqrt{1+9} = \sqrt{10}$

$|\bar{b}| = \sqrt{(2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13}$

$\therefore |\bar{a}| + |\bar{b}| = \sqrt{10} + \sqrt{13}$

7> If $\bar{a} = 2i-3j$, $\bar{b} = 3j-4k$ and $\bar{c} = 4k-2i$ then

$\bar{a}+\bar{b}+\bar{c} = \underline{0}$

Here $a = (2, -3, 0)$, $b = (0, 3, -4)$, $c = (-2, 0, 4)$

$\therefore \bar{a}+\bar{b}+\bar{c} = (2, -3, 0) + (0, 3, -4) + (-2, 0, 4)$

$= (2+0-2, -3+3+0, 0-4+4)$

$= (0, 0, 0)$

$= 0$ (zero vector)

8> If $\bar{a} = 2i+j$ and $\bar{b} = i-3k$ then $\bar{a}\cdot\bar{b} = \underline{2.}$

Here $\bar{a} = (2, 1, 0)$, $\bar{b} = (1, 0, -3)$

Then $\bar{a}\cdot\bar{b} = (2, 1, 0)\cdot(1, 0, -3)$

$= 2+0+0$

$= 2.$

$\therefore \boxed{\bar{a}\cdot\bar{b} = 2}$

9> If $\bar{a} = 2i+j+k$ and $\bar{b} = i-j+3k$ then $\bar{a}\cdot\bar{b} = \underline{4}$

$\bar{a} = (2, 1, 1)$, $\bar{b} = (1, -1, 3)$

Then $\bar{a}\cdot\bar{b} = (2, 1, 1)\cdot(1, -1, 3)$

$= 2-1+3$

$= 4$

$\therefore \boxed{\bar{a}\cdot\bar{b} = 4}$

10> If $\bar{a} = 2i - 2j + k$ and $\bar{b} = i + 3j + 3k$ then $\bar{a} \cdot \bar{b} = \underline{-1}$

Here $\bar{a} = (2, -2, 1)$

$\bar{b} = (1, 3, 3)$.

Then $\bar{a} \cdot \bar{b} = (2, -2, 1) \cdot (1, 3, 3)$

$= 2 - 6 + 3$

$\therefore \boxed{\bar{a} \cdot \bar{b} = -1}$

11> If $\bar{a} = 2i + 3j - k$ and $\bar{b} = 4i + 6j - 2k$ then $\bar{a} \times \bar{b} = \underline{0}$

$\rightarrow$ Here $a = (2, 3, -1)$ & $\bar{b} = (4, 6, -2)$

Then

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 4 & 6 & -2 \end{vmatrix}$$

$= i[-6+6] - j[-4+4] + k[12-12]$

$= 0i - j0 + 0.k$

$\boxed{\bar{a} \times \bar{b} = 0}$ (zero vector)

12> If $\bar{a} = i + 3j - k$, $\bar{b} = 4i + j - 2k$ then $\bar{a} \times \bar{b} = \underline{-(5i + 2j + 11k)}$.

Here $\bar{a} = (1, 3, -1)$

$\bar{b} = (4, 1, -2)$

Then

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 1 & 3 & -1 \\ 4 & 1 & -2 \end{vmatrix}$$

$= i(-6+1) - j(-2+4) + k(1-12)$

$= i(-5) - j(2) + k(-11)$

$= (-5, -2, -11)$.

$= -(5i + 2j + 11k)$

13> If $\bar{a} = i + j$ and $\bar{b} = j - i$ then Angle $(\bar{a}, \bar{b}) = \underline{\pi/2}$

Here $\bar{a} = (1, 1)$ & $\bar{b} = (-1, 1)$.

$|\bar{a}| = \sqrt{1+1} = \sqrt{2}$, $|\bar{b}| = \sqrt{1+1} = \sqrt{2}$

Then $\bar{a} \cdot \bar{b} = (1, 1) \cdot (-1, 1)$

$= -1 + 1$

$= 0$

$\therefore \bar{a} \cdot \bar{b} = 0.$

$\therefore \cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}|\,|\bar{b}|} = \dfrac{0}{\sqrt{2}\,\sqrt{2}} = 0$

$\therefore \theta = \cos^{-1} 0 \Rightarrow \boxed{\theta = \pi/2}$

14> If $\bar{a} = 2i - 3j$, $\bar{b} = i - 3j$ and $\bar{c} = 3i + j$ then $2\bar{a} - (\bar{b} + \bar{c}) =$ $\underline{0 - 4j}$.

Here $a = (2, -3)$, $b = (1, -3)$, $c = (3, 1)$

Now, $2\bar{a} - (\bar{b} + \bar{c})$

$= 2(2, -3) - [(1, -3) + (3, 1)]$

$= (4, -6) - [(4, -2)]$

$= (4 - 4, -6 + 2)$

$= (0, -4)$

$2\bar{a} - (\bar{b} + \bar{c}) = 0 - 4j$

15> If $\bar{a} = i + 3j$ and $\bar{b} = 5i - j$ then $|\bar{a} + 3\bar{b}| =$ $\underline{16}$

Here $a = (1, 3)$, $b = (5, -1)$

$\bar{a} + 3\bar{b} = (1, 3) + 3(5, -1)$

$= (1, 3) + (15, -3)$

$= (16, 0)$

$\therefore |\bar{a} + 3\bar{b}| = \sqrt{(16)^2 + 0} = \sqrt{256}$

$\therefore |\bar{a} + 3\bar{b}| = 16$

16> If $\bar{x} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ then $|\bar{x}| = $ ____

Here $\bar{x} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Then $|\bar{x}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$= \sqrt{1/2 + 1/2} = \sqrt{2/2} = \sqrt{1}$

$\therefore \boxed{|\bar{x}| = 1}$

17> $\bar{a} \times \bar{a} = \underline{\ 0\ }$ (zero vector)

Let $\bar{a} = (1, 1, 1)$

Then

$$\bar{a} \times \bar{a} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= i(1-1) - j(1-1) + k(1-1)$

$= 0i - 0j + 0k$

$= (0, 0, 0)$

$\therefore \boxed{\bar{a} \times \bar{a} = 0}$

18> $\bar{a} \cdot \bar{a} = \underline{|\bar{a}|^2}$

19> $\bar{a} \cdot (\bar{a} \times \bar{b}) = \underline{\ 0\ }$

suppose $\bar{a} = (1, 2, 3)$ & $\bar{b} = (3, 1, 2)$.

Then

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix}$$

$= i(4-3) - j(2-9) + k(1-6)$

$\bar{a} \times \bar{b} = i + 7j - 5k$

$\bar{a} \times \bar{b} = (1, 7, -5)$

$\bar{a}\cdot(\bar{a}\times\bar{b}) = (1,2,3)\cdot(1,7,-5)$

$= 1+14-15 = 0$

$\therefore \boxed{\bar{a}\cdot(\bar{a}\times\bar{b}) = 0}$

20> $(\bar{a}\times\bar{b})\cdot(\bar{b}\times\bar{a}) = \underline{|\bar{a}\times\bar{b}|^2}$

$\rightarrow \quad \bar{b}\times\bar{a} = -(\bar{a}\times\bar{b})$.

$(\bar{a}\times\bar{b})\cdot(\bar{b}\times\bar{a}) = -(\bar{a}\times\bar{b})\cdot(\bar{a}\times\bar{b})$

$= |\bar{a}\times\bar{b}|^2$

* <u>Question for 3 Marks</u>

1> If $\bar{a} = 2i+j-3k$, $\bar{b} = 4i+5j+4k$ and $\bar{c} = 3i-2j+k$

then find $3\bar{a}+2\bar{b}-3\bar{c}$.

$\rightarrow$ Here $\bar{a} = (2,1,-3)$, $\bar{b} = (4,5,4)$, $\bar{c} = (3,-2,1)$

Then $3\bar{a}+2\bar{b}-3\bar{c}$

$= 3(2,1,-3)+2(4,5,4)-3(3,-2,1)$

$= (6,3,-9)+(8,10,8)-(9,-6,3)$

$= (6+8-9,\ 3+10+6,\ -9+8-3)$

$= (5,19,-4)$

$3\bar{a}+2\bar{b}-3\bar{c} = 5i+19j-4k$.

2> If $\bar{a} = i-2j+4k$, $\bar{b} = -3i+j-4k$ and $\bar{c} = i+2j-4k$

then find $|5\bar{a}+3\bar{b}+2\bar{c}|$.

$\rightarrow$ Here $\bar{a} = (1,-2,4)$, $\bar{b} = (-3,1,-4)$, $\bar{c} = (1,2,-4)$

Then $5\bar{a}+3\bar{b}+2\bar{c}$

$= 5(1,-2,4)+3(-3,1,-4)+2(1,2,-4)$

$= (5,-10,20)+(-9,3,-12)+(2,4,-8)$

$= (5-9+2,\ -10+3+4,\ 20-12-8)$

$= (-2, -3, 0)$

$\rightarrow |5\bar{a}+3\bar{b}+2\bar{c}| = \sqrt{(-2)^2+(-3)^2+0^2}$

$= \sqrt{4+9}$

$\therefore \boxed{|5\bar{a}+3\bar{b}+2\bar{c}| = \sqrt{13}}$

3) If $\bar{a} = j+k-i$ and $\bar{b} = 2i+j-3k$ then find $|2\bar{a}+3\bar{b}|$.

$\rightarrow$ Here $\bar{a} = (-1, 1, 1)$

$\bar{b} = (2, 1, -3)$

Then $2\bar{a}+3\bar{b} = 2(-1,1,1)+3(2,1,-3)$

$= (-2,2,2)+(6,3,-9)$

$= (-2+6,\ 2+3,\ 2-9)$

$= (4, 5, -7)$

$\therefore |2\bar{a}+3\bar{b}| = \sqrt{4^2+5^2+(-7)^2}$

$= \sqrt{16+25+49}$

$= \sqrt{90}$

$\therefore |2\bar{a}+3\bar{b}| = 3\sqrt{10}$

4) If $\bar{a} = 3i-j-4k$, $\bar{b} = -2i+4j-3k$ and $\bar{c} = -i+2j-5k$ then find direction cosines of $\bar{a}+2\bar{b}-\bar{c}$.

$\rightarrow$ Here $\bar{a} = (3,-1,-4)$, $\bar{b} = (-2,4,-3)$, $\bar{c} = (-1,2,-5)$

Then $\bar{a}+2\bar{b}-\bar{c}$

$= (3,-1,-4)+2(-2,4,-3) - (-1,2,-5)$

$= (3,-1,-4)+(-4,8,-6) - (-1,2,-5)$

$= (3-4+1,\ -1+8-2,\ -4-6+5)$

$= (0, 5, -5)$

$\therefore |\bar{a} + 2\bar{b} - \bar{c}| = \sqrt{0 + 5^2 + (-5)^2}$

$= \sqrt{0 + 25 + 25}$

$= \sqrt{50}$

$\therefore |\bar{a} + 2\bar{b} - \bar{c}| = 5\sqrt{2}.$

If l, m and n are the direction cosines of $\bar{a} + 2\bar{b} - \bar{c}$ then

$l = \frac{0}{5\sqrt{2}}$, $m = \frac{\cancel{5}}{\cancel{5}\sqrt{2}}$, $n = \frac{-\cancel{5}}{\cancel{5}\sqrt{2}}$

$\therefore l = 0$, $m = 1/\sqrt{2}$, $n = -1/\sqrt{2}$.

5> If $a(1,0,0) + b(0,2,0) + c(0,0,3) = (3,4,9)$ then find a, b and c.

$\rightarrow$ $a(1,0,0) + b(0,2,0) + c(0,0,3) = (3,4,9)$

$\therefore (a,0,0) + (0,2b,0) + (0,0,3c) = (3,4,9)$

$\therefore (a+0+0,\ 0+2b+0,\ 0+0+3c) = (3,4,9)$

$\therefore (a, 2b, 3c) = (3,4,9)$

$\therefore a = 3$, $2b = 4$, $3c = 9$

$\therefore a = 3$, $b = 2$, $c = 3$

6>. If $\bar{a} = 3i - 2j - \sqrt{5}k$ and $\bar{b} = 4i + 2j + \sqrt{5}k$ then find the projection of $\bar{a}$ on $\bar{b}$.

$\rightarrow$ Here $\bar{a} = (3, -2, -\sqrt{5})$, $\bar{b} = (4, 2, \sqrt{5})$

Now, $\bar{a} \cdot \bar{b} = (3, -2, -\sqrt{5}) \cdot (4, 2, \sqrt{5})$

$= 12 - 4 - 5$

$\therefore \bar{a} \cdot \bar{b} = 3.$

$\rightarrow$ $|\bar{b}| = \sqrt{(4)^2+(2)^2+(\sqrt{5})^2}$

$= \sqrt{16+4+5}$

$= \sqrt{25}$

$\therefore |\bar{b}| = 5$

$\rightarrow$ projection of $\bar{a}$ on $\bar{b} = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{b}|} = \dfrac{3}{5}$

7> If $\bar{a} = i-j+k$, $\bar{b} = 2i-j+k$ and $\bar{c} = i+j-2k$ then find $\bar{a}\cdot(\bar{b}+\bar{c})$.

$\rightarrow$ Here $\bar{a} = (1,-1,1)$, $\bar{b} = (2,-1,1)$, $\bar{c} = (1,1,-2)$

Now $\bar{b}+\bar{c} = (2,-1,1)+(1,1,-2)$

$= (3,0,-1)$

$\therefore \bar{a}\cdot(\bar{b}+\bar{c}) = (1,-1,1)\cdot(3,0,-1)$

$= 3-0-1$

$\therefore \bar{a}\cdot(\bar{b}+\bar{c}) = 2$

8> If $\bar{x} = 3i-j+2k$ and $\bar{y} = 2i+j-k$ then find the vector perpendicular to both $\bar{x}$ and $\bar{y}$.

$\rightarrow$ Here $\bar{x} = (3,-1,2)$, $\bar{y} = (2,1,-1)$

$$\bar{x}\times\bar{y} = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

$= i(1-2) - j(-3-4) + k(3+2)$

$= -i + 7j + 5k$

$= (-1,7,5)$

$\therefore |\bar{x}\times\bar{y}| = \sqrt{(-1)^2+7^2+5^2}$

$= \sqrt{1+49+25}$

$\therefore |\bar{x}\times\bar{y}| = \sqrt{75}$

→ Unit vector perpendicular to both $\bar{x}$ & $\bar{y}$ is

$$\frac{\bar{x} \times \bar{y}}{|\bar{x} \times \bar{y}|} = \frac{(-1, 7, 5)}{\sqrt{75}}$$

$$= \left(\frac{-1}{\sqrt{75}}, \frac{7}{\sqrt{75}}, \frac{5}{\sqrt{75}} \right)$$

9> If $\bar{a} = 10i + 2j + 3k$, $\bar{b} = i - 2j + 2k$ and $\bar{c} = 3i - 2j - 2k$

then find $\bar{a} \cdot (\bar{b} \times \bar{c})$.

→ Here $\bar{a} = (10, 2, 3)$, $\bar{b} = (1, -2, 2)$, $\bar{c} = (3, -2, -2)$

$$\bar{b} \times \bar{c} = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$$= i(4+4) - j(-2-6) + k(-2+6)$$

$$= 8i + 8j + 4k$$

$$\therefore \bar{a} \cdot (\bar{b} \times \bar{c}) = (10, 2, 3) \cdot (8, 8, 4)$$

$$= 80 + 16 + 12$$

$$\therefore \bar{a} \cdot (\bar{b} \times \bar{c}) = 108$$

10>. For what value of m, the vectors $2i - 3j + 5k$ and $mi - 6j - 8k$ are perpendicular to each other?

→ Let $\bar{a} = (2, -3, 5)$, $\bar{b} = (m, -6, -8)$

Here $\bar{a}$ & $\bar{b}$ vectors are perpendicular to each other.

$$\therefore \bar{a} \cdot \bar{b} = 0$$

$$\therefore (2, -3, 5) \cdot (m, -6, -8) = 0$$

$$\therefore (2)(m) + (-3)(-6) + (5)(-8) = 0$$

$$\therefore 2m + 18 - 40 = 0$$

$$\therefore 2m - 22 = 0$$

$\therefore\ 2m = 22$

$\therefore\ \boxed{m = 11}$

11) For $\bar{x} = (-4, 9, 6)$, $\bar{y} = (0, 7, 10)$ and $\bar{z} = (-1, 6, 6)$

show that $(\bar{x} - \bar{z}) \cdot (\bar{y} - \bar{z}) = 0$.

→ Here $\bar{x} = (-4, 9, 6)$, $\bar{y} = (0, 7, 10)$, & $\bar{z} = (-1, 6, 6)$.

$\bar{x} - \bar{z} = (-4, 9, 6) - (-1, 6, 6)$

$= (-4+1, 9-6, 6-6)$

$= (-3, 3, 0)$

$\bar{y} - \bar{z} = (0, 7, 10) - (-1, 6, 6)$

$= (0+1, 7-6, 10-6)$

$= (1, 1, 4)$

Now, $(\bar{x} - \bar{z}) \cdot (\bar{y} - \bar{z}) = (-3, 3, 0) \cdot (1, 1, 4)$

$= -3 + 3 + 0$

$(\bar{x} - \bar{z}) \cdot (\bar{y} - \bar{z}) = 0$.

12) show that the angle between the vectors $2i + j + 4k$ and $i + j + k$ is $\cos^{-1} \frac{\sqrt{7}}{3}$.

→ let $\bar{a} = 2i + j + 4k$ & $\bar{b} = i + j + k$

$\bar{a} = (2, 1, 4)$ & $\bar{b} = (1, 1, 1)$

$\therefore |\bar{a}| = \sqrt{(2)^2 + 1 + 4^2}$

$= \sqrt{4 + 1 + 16}$

$= \sqrt{21}$

$|\bar{b}| = \sqrt{1 + 1 + 1}$

$\therefore |\bar{b}| = \sqrt{3}$

$$\bar{a} \cdot \bar{b} = (2, 1, 4) \cdot (1, 1, 1)$$

$$= 2 + 1 + 4$$

$$\therefore \bar{a} \cdot \bar{b} = 7$$

If θ is the angle between $\bar{a}$ & $\bar{b}$ then

$$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| \cdot |\bar{b}|}$$

$$= \frac{7}{\sqrt{21} \cdot \sqrt{3}}$$

$$= \frac{\sqrt{7} \cdot \cancel{\sqrt{7}}}{\cancel{\sqrt{7}}\sqrt{3}\sqrt{3}}$$

$$= \frac{\sqrt{7}}{3}$$

$$\therefore \theta = \cos^{-1}\frac{\sqrt{7}}{3}$$

13> show that the angle between the vectors $i + j - k$ and $2i - 2j + k$ is $\sin^{-1}\sqrt{\frac{26}{27}}$

→ Let $\bar{a} = i + j - k$ & $\bar{b} = 2i - 2j + k$

$\bar{a} = (1, 1, -1)$ $\qquad \bar{b} = (2, -2, 1)$

Then

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & -2 & 1 \end{vmatrix}$$

$$= i(1-2) - j(1+2) + k(-2-2)$$

$$= -i - 3j - 4k$$

$$\therefore |\bar{a} \times \bar{b}| = \sqrt{(-1)^2 + (-3)^2 + (-4)^2}$$

$$= \sqrt{1 + 9 + 16} = \sqrt{26}.$$

$$\therefore \boxed{|\bar{a} \times \bar{b}| = \sqrt{26}}$$

$\therefore |\bar{a}| = \sqrt{1+1+1}$

$= \sqrt{3}$

$|\bar{b}| = \sqrt{(2)^2+(-2)^2+1^2}$

$= \sqrt{4+4+1}$

$|\bar{b}| = \sqrt{9}$

It θ is the angle between two vectors $\bar{a}$ & $\bar{b}$ is

$$\sin\theta = \frac{|\bar{a}\times\bar{b}|}{|\bar{a}|\,|\bar{b}|}$$

$$= \frac{\sqrt{26}}{\sqrt{3}\,\sqrt{9}}$$

$$\therefore \theta = \sin^{-1}\frac{\sqrt{26}}{\sqrt{27}}$$

$$\therefore \theta = \sin^{-1}\sqrt{\frac{26}{27}}$$

14> Find a unit vector perpendicular to each other $\bar{a} = (5, 7, -2)$ and $\bar{b} = (3, 1, -2)$.

$\rightarrow$

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 5 & 7 & -2 \\ 3 & 1 & -2 \end{vmatrix}$$

$$= i(-14+2) - j(-10+6) + k(5-21)$$

$$= -12i + 4j - 16k$$

$$\therefore |\bar{a}\times\bar{b}| = \sqrt{(-12)^2+(4)^2+(-16)^2}$$

$$= \sqrt{144+16+256}$$

$$|\bar{a}\times\bar{b}| = \sqrt{416} = 4\sqrt{26}$$

- Unit vector perpendicular to bot $\bar{a}$ & $\bar{b}$ is

$$\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}$$

$$= \frac{(-12, 4, -16)}{4\sqrt{26}}$$

$$= \frac{4(-3, 1, -4)}{4\sqrt{26}}$$

$$= \frac{(-3, 1, -4)}{\sqrt{26}}$$

* <u>Question</u> <u>for</u> <u>4</u> <u>Marks</u>

1). If $\bar{x} = (1,1,1)$ and $\bar{y} = (2,-1,-1)$ then P. T. $\bar{x}$ is perpendicular to $\bar{y}$. Also find an unit vector perpendicular to both $\bar{x}$ and $\bar{y}$.

→ Here $\bar{x} = (1,1,1)$ & $\bar{y} = (2,-1,-1)$.

Now, $\bar{x}\cdot\bar{y} = (1,1,1)\cdot(2,-1,-1)$

$= 2 - 1 - 1$

$\therefore \bar{x}\cdot\bar{y} = 0$

so, $\bar{x}$ is perpendicular to $\bar{y}$.

$$\bar{x}\times\bar{y} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix}$$

$= i(-1+1) - j(-1-2) + k(-1-2)$

$= 0i + 3j - 3k.$

$= (0, 3, -3).$

$|\bar{x}\times\bar{y}| = \sqrt{0+9+9} = \sqrt{18} = 3\sqrt{2}.$

→ Unit vector perpendicular to both $\bar{x}$ & $\bar{y}$ is $\dfrac{\bar{x}\times\bar{y}}{|\bar{x}\times\bar{y}|}$

$$= \frac{(0,3,-3)}{3\sqrt{2}}$$

$$= \frac{(0,1,-1)}{\sqrt{2}}$$

2) If $\bar{a} = 2i - 3j + 4k$ and $\bar{b} = i - j + k$ then find Unit vector perpendicular to $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$.

→ Here $\bar{a} = (2,-3,4)$ and $\bar{b} = (1,-1,1)$

Then $\bar{a}+\bar{b} = (2,-3,4) + (1,-1,1)$

$= (3,-4,5)$

And $\bar{a}-\bar{b} = (2,-3,4) - (1,-1,1)$

$= (2-1, -3+1, 4-1)$

$= (1,-2,3)$

Unit vector perpendicular to $\bar{a}+\bar{b}$ & $\bar{a}-\bar{b}$ is

$$\frac{(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})}{|(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})|}$$

$$\to (\bar{a}+\bar{b})\times(\bar{a}-\bar{b}) = \begin{vmatrix} i & j & k \\ 3 & -4 & 5 \\ 1 & -2 & 3 \end{vmatrix}$$

$= i(-12+10) - j(9-5) + k(-6+4)$

$= -2i - 4j - 2k$

$= (-2,-4,-2)$

$|(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})| = \sqrt{4+16+4}$

$= \sqrt{24}$

$= 2\sqrt{6}.$

Unit vector perpendicular to $\bar{a}+\bar{b}$ & $\bar{a}-\bar{b}$ is $\dfrac{(-2,-4,-2)}{2\sqrt{6}}$

$$= \frac{(-1,-2,-1)}{\sqrt{6}}$$

3> A body is acted upon the forces $3i-2j+k$ and $-i-j+2k$. If the body moves under the forces from the point $(2,2,-3)$ to $(-1,2,4)$ then find work-done.

→ Let $\bar{F}_1 = (3,-2,1)$, & $\bar{F}_2 = (-1,-1,2)$

Resultant force $\bar{F}_R = \bar{F}_1 + \bar{F}_2$

$= (3,-2,1) + (-1,-1,2)$

$= (2,-3,3)$

$d_1 = (2,2,-3)$, $d_2 = (-1,2,4)$

displacement $d = d_2 - d_1$

$= (-1,2,4) - (2,2,-3)$

$= (-3,0,7)$

work done $W = F_R \cdot d$

$= (2,-3,3) \cdot (-3,0,7)$

$= -6 + 0 + 21$

$\therefore \boxed{W = 15 \text{ Unit}}$

4> A body is acted upon the forces $3i-2j+3k$ and $-j+2k$. If the body moves under the forces from the point $(2,0,-3)$ to $(-1,2,2)$ then find work done.

→ Let $\bar{F}_1 = (3,-2,3)$, $\bar{F}_2 = (0,-1,2)$

$d_1 = (2,0,-3)$ $d_2 = (-1,2,2)$.

Resultant Force $F = \bar{F}_1 + \bar{F}_2$

$= (3,-2,3) + (0,-1,2)$

$= (3,-3,5)$

displacement $d = d_2 - d_1$

$= (-1,2,2) - (2,0,-3)$

$= (-3,2,5)$

→ Total work done $W = F \cdot d$.

$= (3, -3, 5) \cdot (-3, 2, 5)$

$= -9 - 6 + 25$

$\therefore \boxed{W = 10 \text{ Unit}}$

5> Forces $3i - j + 2k$ and $i + 3j - k$ are acting on a particle and the particle moves from $2i + 3j + k$ to the point $5i + 2j + 3k$ under these forces. Find the work done by the force.

→ Here $\bar{F}_1 = (3, -1, 2)$, $\bar{F}_2 = (1, 3, -1)$

$d_1 = (2, 3, 1)$, $d_2 = (5, 2, 3)$.

Resultant Force $\bar{F}_R = \bar{F}_1 + \bar{F}_2$

$= (3, -1, 2) + (1, 3, -1)$

$= (4, 2, 1)$

displacement $d = d_2 - d_1$

$= (5, 2, 3) - (2, 3, 1)$

$= (3, -1, 2)$

work done $W = F_R \cdot d$.

$= (4, 2, 1) \cdot (3, -1, 2)$

$= 12 - 2 + 2$

$= 12$

$\therefore \boxed{W = 12 \text{ Unit}}$

6> A particle moves ~~from~~ from the point $3i - 2j + k$ to the point $i + 3j - 4k$ under the effect of constant forces $i - j + k$, $i + j - 3k$ and $4i + 5j - 6k$. Find the work done.

→ Let $F_1 = (1, -1, 1)$, $F_2 = (1, 1, -3)$, $F_3 = (4, 5, -6)$

Resultant Forces $F_R = \bar{F_1} + \bar{F_2} + \bar{F_3}$

$= (1, -1, 1) + (1, 1, -3) + (4, 5, -6)$

$= (1 + 1 + 4, -1 + 1 + 5, 1 - 3 - 6)$

$= (6, 5, -8)$

displacement $d = (1, 3, -4) - (3, -2, 1)$

$= (1 - 3, 3 + 2, -4 - 1)$

$d = (-2, 5, -5)$

Work done $w = F_R \cdot d$

$= (6, 5, -8) \cdot (-2, 5, -5)$

$= -12 + 25 + 40$

$\boxed{W = 53 \text{ Unit}}$

7) A Force $F = 2i + j + k$ is acting at the point $(-3, 2, 1)$. Find the Magnitude of the moments of Force F about the point $(2, 1, 2)$.

→ Here $\bar{F} = (2, 1, 1)$.

Let $A(2, 1, 2)$ and $P(-3, 2, 1)$ be the given points.

$\therefore AP = OP - OA$

$= (-3, 2, 1) - (2, 1, 2)$

$= (-3 - 2, 2 - 1, 1 - 2)$

$AP = (-5, 1, -1)$

Moment of the force F about the point $A = \bar{AP} \times \bar{F}$

$= (-5, 1, -1) \times (2, 1, 1)$

$$= \begin{vmatrix} i & j & k \\ -5 & 1 & -1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= i(1+1) - j(-5+2) + k(-5-2)$$

$$= 2i + 3j - 7k$$

$\overline{AP} \times \overline{F} = (2, 3, -7)$

Magnitude of the moment $= \sqrt{(2)^2+(3)^2+(-7)^2}$

$= \sqrt{4+9+49}$

$= \sqrt{62}.$

8) Find the moment about the point $(2,3,-1)$ of the force $3i-k$ acting through the point $(1,-2,1)$. Also find the magnitude of the moment.

→ Let $\overline{F} = 3i - k$, $\overline{F} = (3, 0, -1)$

Let $A(2,3,-1)$ and $P(1,-2,1)$ be the given points.

$\therefore AP = OP - OA$

$= (1,-2,1) - (2,3,-1)$

$= (1-2, -2-3, 1+1)$

$= (-1, -5, 2).$

Moment of the Force F about the point $A = \overline{AP} \times \overline{F}$

$= (-1,-5,2) \times (3,0,-1)$

$$= \begin{vmatrix} i & j & k \\ -1 & -5 & 2 \\ 3 & 0 & -1 \end{vmatrix}$$

$= i(5-0) - j(1-6) + k(0+15) = 5i + 5j + 15k$

$= (5, 5, 15)$

$\therefore$ Magnitude of the moment $= |\overline{AP} \times \overline{F}|$

$= \sqrt{(5)^2+(5)^2+(15)^2}$

$= \sqrt{25+25+225} = \sqrt{275} = 5\sqrt{11}$