BASIC MATHEMATICS.
(3300001)

* If $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix}$ then find $2A - 4B + C$.

$\Rightarrow$ $2A - 4B + C$

$$= 2\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 7 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} -1 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 & 2 \\ 0 & 2 & 0 \\ 14 & 16 & 18 \end{bmatrix} - \begin{bmatrix} -4 & -8 & 0 \\ 4 & 4 & -4 \\ 8 & 8 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+4+3 & 4+8+0 & 2-0+5 \\ 0-4+6 & 2-4+9 & 0+4-1 \\ 14-8+7 & 16-8+8 & 18-8-2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 12 & 7 \\ 2 & 7 & 3 \\ 13 & 16 & 8 \end{bmatrix}.$$

* If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$ then find 'X' from $X + A + B = 0$.

$\Rightarrow$ $X + A + B = 0$

$\Rightarrow (-X) = A + B$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$$

$$\Rightarrow (-X) = \begin{bmatrix} 4 & 0 & 5 \\ 4 & 9 & 2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -4 & 0 & -5 \\ -4 & -9 & -2 \end{bmatrix}.$$

* If $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & -4 \\ 5 & 1 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 17 & -1 & 3 \\ -24 & -1 & -16 \\ -7 & 1 & 1 \end{bmatrix}$ and $4A + 3C = B$ then find matrix 'C'.

$\Rightarrow$ $4A + 3C = B$

$\Rightarrow$ $3C = B - 4A$

$$= \begin{bmatrix} 17 & -1 & 3 \\ -24 & -1 & -16 \\ -7 & 1 & 1 \end{bmatrix} - 4\begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & -4 \\ 5 & 1 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & -1 & 3 \\ -24 & -1 & -16 \\ -7 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 8 & -4 & 0 \\ 12 & 8 & -16 \\ 20 & 4 & 36 \end{bmatrix}$$

$$= \begin{bmatrix} 17-8 & -1+4 & 3-0 \\ -24-12 & -1-8 & -16+16 \\ -7-20 & 1-4 & 1-36 \end{bmatrix}$$

$$\Rightarrow 3C = \begin{bmatrix} 9 & 3 & 3 \\ -36 & -9 & 0 \\ -27 & -3 & -35 \end{bmatrix}$$

$$\Rightarrow C = \frac{1}{3}\begin{bmatrix} 9 & 3 & 3 \\ -36 & -9 & 0 \\ -27 & -3 & -35 \end{bmatrix}.$$

* If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ then prove that $(A+B)^T = A^T + B^T$.

$$\Rightarrow A + B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & 4-2 \\ 1+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$$

$$\Rightarrow (A+B)^T = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} \text{ ——— } (1)$$

Now, $A^T = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$, $B^T = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$

$$A^T + B^T = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow A^T + B^T = \begin{bmatrix} 3-1 & 1+2 \\ 4-2 & 2+1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} \text{ ——— } (2)$$

from eqⁿ (1) & (2) $(A+B)^T = A^T + B^T$.

* If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$ then find AB and BA.

$$* \Rightarrow AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+3 & 2+2+6 \\ 4+10+6 & 8+5+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 10 \\ 20 & 25 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 2+4 & 4+5 & 6+6 \\ 1+8 & 2+10 & 3+12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 & 15 \\ 6 & 9 & 12 \\ 9 & 12 & 15 \end{bmatrix}.$$

* If $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix}$ then Prove that $(AB)^T = B^T \cdot A^T$.

$$\Rightarrow AB = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2-8 & 10+6 \\ -3+4 & 15-3 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -10 & 16 \\ 1 & 12 \end{bmatrix}$$

$$\Rightarrow (AB)^T = \begin{bmatrix} -10 & 1 \\ 16 & 12 \end{bmatrix} \quad — (1)$$

Now, $B^T = \begin{bmatrix} -1 & 4 \\ 5 & -3 \end{bmatrix}$, $A^T = \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix}$

$$\Rightarrow B^T \cdot A^T = \begin{bmatrix} -1 & 4 \\ 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2-8 & -3+4 \\ 10+6 & 15-3 \end{bmatrix}$$

$$\Rightarrow B^T \cdot A^T = \begin{bmatrix} -10 & 1 \\ 16 & 12 \end{bmatrix} \quad — (2)$$

from eq^n (1) & (2) $(AB)^T = B^T \cdot A^T$.

* If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then prove that

$$A^2 - 5A - 2I = 0.$$

$$\Rightarrow A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}.$$

$$L.H.S = A^2 - 5A - 2I$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7-5-2 & 10-10-0 \\ 15-15-0 & 22-20-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = R.H.S.$$

* If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then prove that

$$A^2 - 4A + 7I = 0$$

$$\Rightarrow A^2 = A \cdot A$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

$$L.H.S = A^2 - 4A + 7I$$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = R.H.S.$$

* If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ then prove that $A^2 = A$

$$\Rightarrow A^2 = A \cdot A$$

$$= \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -3-9+15 & -5-15+25 \\ -1-3+5 & 3+9-15 & 5+15-25 \\ 1+3-5 & -3-9+15 & -5-15+25 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} = A.$$

* If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then Prove that $Adj(A) = A$.

$\Rightarrow$ Far $Adj(A)$

$A_{11} = \begin{vmatrix} 0 & 1 \\ 4 & 3 \end{vmatrix} = 0-4 = -4$ $\quad A_{12} = \begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 3-4 = -1$

$A_{13} = \begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = 4-0 = 4$ $\quad A_{21} = \begin{vmatrix} -3 & -3 \\ 4 & 3 \end{vmatrix} = -9+12 = 3$

$A_{22} = \begin{vmatrix} -4 & -3 \\ 4 & 3 \end{vmatrix} = -12+12 = 0$ $\quad A_{23} = \begin{vmatrix} -4 & -3 \\ 4 & 4 \end{vmatrix} = -16+12 = -4$

$A_{31} = \begin{vmatrix} -3 & -3 \\ 0 & 1 \end{vmatrix} = -3+0 = -3$ $\quad A_{32} = \begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix} = -4+3 = -1$

$A_{33} = \begin{vmatrix} -4 & -3 \\ 1 & 0 \end{vmatrix} = 0+3 = 3$

$\Rightarrow$ matrix $= \begin{bmatrix} -4 & -1 & 4 \\ 3 & 0 & -4 \\ -3 & -1 & 3 \end{bmatrix}$ $\Rightarrow$ matrix of co-factor $= \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}$

$\Rightarrow Adj(A) = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = A.$

* Find the inverse of $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ Let $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ 5 & 1 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 \\ 5 & 0 \end{vmatrix}$

$= 3(1+0) + 1(4+5) + 2(0-5)$

$= 3+9-10 = 2 \neq 0$

$\Rightarrow A^{-1}$ is exists.

$\Rightarrow$ Far $Adj(A)$

$A_{11} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1+0 = 1$ $\quad A_{12} = \begin{vmatrix} 4 & -1 \\ 5 & 1 \end{vmatrix} = 4+5 = 9$

$A_{13} = \begin{vmatrix} 4 & 1 \\ 5 & 0 \end{vmatrix} = 0-5 = -5$ $\quad A_{21} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1-0 = -1$

$A_{22} = \begin{vmatrix} 3 & 2 \\ 5 & 1 \end{vmatrix} = 3-10 = -7$ $\quad A_{23} = \begin{vmatrix} 3 & -1 \\ 5 & 0 \end{vmatrix} = 0+5 = 5$

$A_{31} = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1-2 = -1$ $\quad A_{32} = \begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} = -3-8 = -11$

$A_{33} = \begin{vmatrix} 3 & -1 \\ 4 & 1 \end{vmatrix} = 3+4 = 7$

$\Rightarrow$ matrix $= \begin{bmatrix} 1 & 9 & -5 \\ -1 & -7 & 5 \\ -1 & -11 & 7 \end{bmatrix}$ $\Rightarrow$ matrix of co-factor $= \begin{bmatrix} 1 & -9 & -5 \\ 1 & -7 & -5 \\ -1 & 11 & 7 \end{bmatrix}$

$\Rightarrow Adj\ A = \begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{|A|} \cdot Adj(A)$

$= \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}.$

* Find the inverse of $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$\Rightarrow$ let $A = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{vmatrix}$

$= -1\begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 4 & 5 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix}$

$= -1(5-0) - 2(10-0) - 3(-4-4)$

$= -5-20+24 = -1 \neq 0$

$\Rightarrow A^{-1}$ is exists.

$\Rightarrow$ Far $Adj(A)$

$A_{11} = \begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix} = 5+0 = 5$ $\quad A_{12} = \begin{vmatrix} 2 & 0 \\ 4 & 5 \end{vmatrix} = 10-0 = 10$

$A_{13} = \begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix} = -4-4 = -8$ $\quad A_{21} = \begin{vmatrix} 2 & -3 \\ -2 & 5 \end{vmatrix} = 10-6 = 4$

$A_{22} = \begin{vmatrix} -1 & -3 \\ 4 & 5 \end{vmatrix} = -5+12 = 7$ $\quad A_{23} = \begin{vmatrix} -1 & 2 \\ 4 & -2 \end{vmatrix} = 2-8 = -6$

$A_{31} = \begin{vmatrix} 2 & -3 \\ 1 & 0 \end{vmatrix} = 0+3 = 3 \qquad A_{32} = \begin{vmatrix} -1 & -3 \\ 2 & 0 \end{vmatrix} = 0+6 = 6$

$A_{33} = \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = -1-4 = -5$

$\Rightarrow$ matrix $= \begin{bmatrix} 5 & 10 & -8 \\ 4 & 7 & -6 \\ 3 & 6 & -5 \end{bmatrix} \Rightarrow$ matrix of co-factor $= \begin{bmatrix} 5 & -10 & -8 \\ -4 & 7 & 6 \\ 3 & -6 & -5 \end{bmatrix}$

$\Rightarrow Adj(A) = \begin{bmatrix} 5 & -4 & 3 \\ -10 & 7 & -6 \\ -8 & 6 & -5 \end{bmatrix}$

$A^{-1} = \dfrac{Adj(A)}{|A|} = \left(-\dfrac{1}{1}\right) \cdot \begin{bmatrix} 5 & -4 & 3 \\ -10 & 7 & -6 \\ -8 & 6 & 5 \end{bmatrix}.$

* If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ then verify that $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

$\Rightarrow A \cdot B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2+1 & 4+3 \\ 0+1 & 0+3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow |AB| = \begin{vmatrix} 3 & 7 \\ 1 & 3 \end{vmatrix} = 9-7 = 2 \neq 0$

$\Rightarrow (AB)^{-1}$ is exists.

$\Rightarrow Adj(AB) = \begin{bmatrix} 3 & -7 \\ -1 & 3 \end{bmatrix}$

$\Rightarrow (AB)^{-1} = \dfrac{Adj(AB)}{|AB|} = \dfrac{1}{2}\begin{bmatrix} 3 & -7 \\ 1 & 3 \end{bmatrix}$ — (1)

* For B^{-1}

$|B| = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$

$= 6-4 = 2 \neq 0$

B^{-1} is exists

$\Rightarrow Adj(B) = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow B^{-1} = \dfrac{Adj(B)}{|B|}$

$= \dfrac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

* For A^{-1}

$|A| = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

$= 1-0 = 1 \neq 0$

A^{-1} is exists

$\Rightarrow Adj(A) = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{Adj(A)}{|A|}$

$= \dfrac{1}{1}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow B^{-1} \cdot A^{-1}$

$= \dfrac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} \cdot \dfrac{1}{1}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$= \dfrac{1}{2}\begin{bmatrix} 3+0 & -3-4 \\ -1+0 & 1+2 \end{bmatrix}$

$B^{-1} \cdot A^{-1} = \dfrac{1}{2}\begin{bmatrix} 3 & -7 \\ -1 & 3 \end{bmatrix}$ — (2)

from eq^n (1) & (2) $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

* Solve the equation $5x+3y=11$ and $3x-2y=-1$ using matrix method.

$\Rightarrow 5x+3y = 11$

$3x-2y = -1$

$\Rightarrow \begin{bmatrix} 5 & 3 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -1 \end{bmatrix}$

$A \cdot X = B$

$\Rightarrow X = A^{-1} \cdot B$ — (1)

* For A^{-1}

$|A| = \begin{vmatrix} 5 & 3 \\ 3 & -2 \end{vmatrix} = -10-9 = -19 \neq 0$

A^{-1} is exists.

$\Rightarrow Adj(A) = \begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix}$

$A^{-1} = \dfrac{Adj(A)}{|A|} = -\dfrac{1}{19}\begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix}$

from (1) $X = A^{-1} \cdot B$

$= -\dfrac{1}{19}\begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 11 \\ -1 \end{bmatrix}$

$= -\dfrac{1}{19}\begin{bmatrix} -22+3 \\ -33-5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\dfrac{1}{19}\begin{bmatrix} -19 \\ -38 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -19/-19 \\ -38/-19 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\Rightarrow x = 1, \; y = 2.$

* Solve the equation $2x+3y=1$ and $y-4x=2$ using matrix method.

$\Rightarrow$ $2x+3y=1$
$-4x+y=2$

$\Rightarrow \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$A \cdot X = B$

$\Rightarrow X = A^{-1} \cdot B$ —(1)

* For A^{-1}

$|A| = \begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix} = 2+12 = 14 \neq 0$

A^{-1} is exists

$\Rightarrow \mathrm{Adj}(A) = \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \cdot \mathrm{Adj}(A) = \frac{1}{14} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$

from (1)

$X = A^{-1} \cdot B$

$= \frac{1}{14} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \frac{1}{14} \begin{bmatrix} 1-6 \\ 4+4 \end{bmatrix}$

$= \frac{1}{14} \begin{bmatrix} -5 \\ 8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5/14 \\ 8/14 \end{bmatrix}$

$\Rightarrow x = -\frac{5}{14}, \quad y = \frac{8}{14}.$

* Solve the equation $3x+2y=5$ and $2x-y=1$ using matrix method.

$\Rightarrow$ $3x+2y=5$
$2x-y=1$

$\Rightarrow \begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$A \cdot X = B$

$\Rightarrow X = A^{-1} \cdot B$ —(1)

$\Rightarrow$ For A^{-1}

$|A| = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -3-4 = -7 \neq 0$

A^{-1} is exists.

$\Rightarrow \mathrm{Adj}(A) = \begin{bmatrix} -1 & -2 \\ -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{\mathrm{Adj}(A)}{|A|} = -\frac{1}{7} \begin{bmatrix} -1 & -2 \\ -2 & 3 \end{bmatrix}$

$\Rightarrow$ from (1)

$X = A^{-1} \cdot B$

$= -\frac{1}{7} \begin{bmatrix} -1 & -2 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$= -\frac{1}{7} \begin{bmatrix} -5-2 \\ -10+3 \end{bmatrix}$

$= -\frac{1}{7} \begin{bmatrix} -7 \\ -7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\Rightarrow x = 1, \quad y = 1$

* Solve the equation $2x+3y=6xy$ and $x-y=xy$ using matrix method.

$2x+3y=6xy, \quad x-y=xy$

$\Rightarrow \frac{2x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy}, \quad \frac{x}{xy} - \frac{y}{xy} = \frac{xy}{xy}$

$\Rightarrow \frac{2}{y} + \frac{3}{x} = 6, \quad \frac{1}{y} - \frac{1}{x} = 1$

Let $\frac{1}{y} = a$ and $\frac{1}{x} = b$

$\Rightarrow 2a+3b=6, \quad a-b=1$

$\Rightarrow$ $2a+3b=6$
$a-b=1$

$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

$A \cdot X = B$

$\Rightarrow X = A^{-1} \cdot B$ —(1)

$\Rightarrow$ For A^{-1}

$|A| = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2-3 = -5 \neq 0$

A^{-1} is exists.

$\Rightarrow Adj(A) = \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix}$

$A^{-1} = -\frac{1}{5}\begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix}$

From ①

$X = A^{-1}.B$

$\begin{bmatrix} a \\ b \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

$= -\frac{1}{5}\begin{bmatrix} -6-3 \\ -6+2 \end{bmatrix}$

$= -\frac{1}{5}\begin{bmatrix} -9 \\ -4 \end{bmatrix}$

$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 9/5 \\ 4/5 \end{bmatrix}$

$\Rightarrow a = 9/5 \quad , \quad b = 4/5$

$\Rightarrow \frac{1}{y} = a = 9/5 \quad , \quad \frac{1}{x} = b = 4/5$

$\Rightarrow y = 5/9 \quad , \quad x = 5/4.$

* If $\bar{a} = j+k-i$, $\bar{b} = 2i+j-3k$ then find $|2\bar{a}+3\bar{b}|$

$\Rightarrow \bar{a} = j+k-i = -i+j+k = (-1,1,1)$

$\bar{b} = 2i+j-3k = (2,1,-3)$

$\Rightarrow 2\bar{a}+3\bar{b} = 2(-1,1,1)+3(2,1,-3)$

$= (-2,2,2)+(6,3,-9)$

$= (-2+6, 2+3, 2-9)$

$= (4,5,-7)$

$\Rightarrow |2\bar{a}+3\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{(4)^2+(5)^2+(-7)^2}$

$= \sqrt{16+25+49}$

$= \sqrt{90} = 3\sqrt{10}.$

* If $\bar{a} = i+2j-k$, $\bar{b} = 3i+j+2k$ and $\bar{c} = -i+2j+2k$ then find $|2\bar{a}-3\bar{b}-5\bar{c}|$

$\Rightarrow \bar{a} = (1,2,-1), \ \bar{b} = (3,1,2), \ \bar{c} = (-1,2,2)$

$\Rightarrow 2\bar{a}-3\bar{b}-5\bar{c} = 2(1,2,-1)-3(3,1,2)-5(-1,2,2)$

$= (2,4,-2)-(9,3,6)-(-5,10,10)$

$= (2-9+5, 4-3-10, -2-6-8)$

$= (-2,-9,-18)$

$\Rightarrow |2\bar{a}-3\bar{b}-5\bar{c}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{(-2)^2+(-9)^2+(-18)^2}$

$= \sqrt{4+81+324}$

$= \sqrt{409}.$

* If $\bar{a} = 3i-2j+k$, $\bar{b} = 2i-4j-3k$ and $\bar{c} = -i+2j+2k$ then find $|2\bar{a}-3\bar{b}-5\bar{c}|$

$\Rightarrow \bar{a} = (3,-2,1), \ \bar{b} = (2,-4,-3), \ \bar{c} = (-1,2,2)$

$\Rightarrow 2\bar{a}-3\bar{b}-5\bar{c} = 2(3,-2,1)-3(2,-4,-3)-5(-1,2,2)$

$= (6,-4,2)-(6,-12,-9)-(-5,10,10)$

$= (6-6+5, -4+12-10, 2+9-10)$

$= (5,-2,1)$

$\Rightarrow |2\bar{a}-3\bar{b}-5\bar{c}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{(5)^2+(-2)^2+(1)^2}$

$= \sqrt{25+4+1}$

$= \sqrt{30}.$

* If $\bar{x} = (-4,9,6)$, $\bar{y} = (0,7,10)$ and $\bar{z} = (-1,6,6)$ then prove that $(\bar{x}-\bar{z})\cdot(\bar{y}-\bar{z}) = 0$

$\Rightarrow (\bar{x}-\bar{z}) = (-4,9,6)-(-1,6,6)$

$= (-4+1, 9-6, 6-0)$

$= (-3,3,0)$

$\Rightarrow (\bar{y}-\bar{z}) = (0,7,10)-(-1,6,6)$

$= (0+1, 7-6, 10-6)$

$= (1,1,4)$

$L.H.S. = (\bar{x}-\bar{z}).(\bar{y}-\bar{z})$

$= (-3,3,0).(1,1,4)$

$= -3+3+0$

$= 0 = R.H.S.$

* If $2\hat{i}-3\hat{j}+5\hat{k}$ and $R\hat{i}-6\hat{j}-8\hat{k}$ are perpendicular to each other then find the value of 'R'.

$\Rightarrow \bar{a} = (2,-3,5)$, $\bar{b} = (R,-6,-8)$

Given that $\bar{a} \perp \bar{b}$, so $\bar{a}.\bar{b} = 0$

$\Rightarrow (2,-3,5).(R,-6,-8) = 0$

$\Rightarrow 2R+18-40 = 0$

$\Rightarrow 2R-22 = 0$

$\Rightarrow 2R = 22$

$\Rightarrow R = \frac{22}{2} = 11.$

* For what value of 'm' the vectors $m\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+5\hat{k}$ are perpendicular to each other?

$\Rightarrow$ Here $\bar{a} = (m,2,1)$, $\bar{b} = (2,4,5)$

Given that $\bar{a} \perp \bar{b}$, so $\bar{a}.\bar{b} = 0$

$\Rightarrow (m,2,1).(2,4,5) = 0$

$\Rightarrow 2m+8+5 = 0$

$\Rightarrow 2m+13 = 0$

$\Rightarrow 2m = -13$

$\Rightarrow m = \frac{-13}{2}.$

* If $2\hat{i}+3\hat{j}-\hat{k}$ and $p\hat{i}-\hat{j}+3\hat{k}$ are perpendicular to each other then find the value of 'p'.

$\Rightarrow$ Here $\bar{a} = (2,3,-1)$, $\bar{b} = (P,-1,3)$

Given that $\bar{a} \perp \bar{b}$, so $\bar{a}.\bar{b} = 0$

$\Rightarrow (2,3,-1).(P,-1,3) = 0$

$\Rightarrow 2P-3-3 = 0$

$\Rightarrow 2P-6 = 0$

$\Rightarrow 2P = 6$

$\Rightarrow P = \frac{6}{2} = 3.$

* Prove that the angle between two vectors $\hat{i}+\hat{j}-\hat{k}$ and $2\hat{i}-2\hat{j}+\hat{k}$ is $\sin^{-1}\left(\sqrt{\frac{26}{27}}\right)$

$\Rightarrow \bar{a} = (1,1,-1)$, $\bar{b} = (2,-2,1)$

$\bar{a}.\bar{b} = (1,1,-1).(2,-2,1)$

$= 2-2-1$

$= -1$

$|\bar{a}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{1+1+1}$

$= \sqrt{3}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{4+4+1}$

$= \sqrt{9}.$

$$\cos\theta = \frac{\bar{a}.\bar{b}}{|\bar{a}|.|\bar{b}|} = \frac{-1}{\sqrt{3}.\sqrt{9}} = \frac{-1}{\sqrt{27}}$$

Now, $\sin^2\theta = 1-\cos^2\theta$

$= 1-\left(\frac{-1}{\sqrt{27}}\right)^2$

$= 1-\frac{1}{27}$

$= \frac{27-1}{27}$

$\Rightarrow \sin^2\theta = \frac{26}{27}$

$\Rightarrow \sin\theta = \sqrt{\frac{26}{27}}$

$\Rightarrow \theta = \sin^{-1}\left(\sqrt{\frac{26}{27}}\right)$

* Prove that the angle between two vectors $\hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+3\hat{k}$ is $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$

$\Rightarrow \bar{a} = (1,2,0)$, $\bar{b} = (1,1,3)$

$\bar{a}.\bar{b} = (1,2,0).(1,1,3)$

$= 1+2+0$

$= 3$

$|\bar{a}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{1+4+0}$

$= \sqrt{5}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{1+1+9}$

$= \sqrt{11}$

$\Rightarrow \cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} = \dfrac{3}{\sqrt{5}\cdot\sqrt{11}} = \dfrac{3}{\sqrt{55}}$

Now $\sin^2\theta = 1-\cos^2\theta$

$= 1-\left(\dfrac{3}{\sqrt{55}}\right)^2$

$= 1-\dfrac{9}{55}$

$= \dfrac{55-9}{55}$

$\Rightarrow \sin^2\theta = \dfrac{46}{55}$

$\Rightarrow \sin\theta = \sqrt{\dfrac{46}{55}}$

$\Rightarrow \theta = \sin^{-1}\left(\sqrt{\dfrac{46}{55}}\right)$

* Prove that the angle between two vectors $3i+j+2k$ and $2i-2j+4k$ is $\sin^{-1}\left(\dfrac{2}{\sqrt{7}}\right)$

$\Rightarrow \bar{a} = (3,1,2),\ \bar{b} = (2,-2,4)$

$\bar{a}\cdot\bar{b} = (3,1,2)\cdot(2,-2,4)$

$= 6-2+8 = 12$

$|\bar{a}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{9+1+4}$

$= \sqrt{14}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{4+4+16}$

$= \sqrt{24}$

$\Rightarrow \cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} = \dfrac{12}{\sqrt{14}\cdot\sqrt{24}} = \dfrac{12}{\sqrt{336}}$

Now, $\sin^2\theta = 1-\cos^2\theta$

$= 1-\left(\dfrac{12}{\sqrt{336}}\right)^2$

$= 1-\dfrac{144}{336}$

$= \dfrac{336-144}{336}$

$= \dfrac{192}{336} = \dfrac{4\times48}{7\times48}$

$\sin^2\theta = \dfrac{4}{7}$

$\Rightarrow \sin\theta = \dfrac{2}{\sqrt{7}}$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{2}{\sqrt{7}}\right).$

* Prove that the angle between two vectors $i+2j-3k$ and $2i+j-k$ is $\sin^{-1}\left(\sqrt{\dfrac{35}{84}}\right)$

$\Rightarrow \bar{a} = (1,2,-3),\ \bar{b} = (2,1,-1)$

$\bar{a}\cdot\bar{b} = (1,2,-3)\cdot(2,1,-1)$

$= 2+2+3$

$= 7$

$|\bar{a}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{4+1+1}$

$= \sqrt{6}$

$\Rightarrow \cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} = \dfrac{7}{\sqrt{14}\cdot\sqrt{6}} = \dfrac{7}{\sqrt{84}}$

Now $\sin^2\theta = 1-\cos^2\theta$

$= 1-\left(\dfrac{7}{\sqrt{84}}\right)^2$

$= 1-\dfrac{49}{84}$

$= \dfrac{84-49}{84}$

$\Rightarrow \sin^2\theta = \dfrac{35}{84}$

$\Rightarrow \sin\theta = \sqrt{\dfrac{35}{84}}$

$\Rightarrow \theta = \sin^{-1}\left(\sqrt{\dfrac{35}{84}}\right)$

* Find the angle between two vectors $(1,2,3)$ and $(-2,3,1)$

$\Rightarrow \bar{a} = (1,2,3),\ \bar{b} = (-2,3,1)$

$\bar{a}\cdot\bar{b} = (1,2,3)\cdot(-2,3,1)$

$= -2+6+3 = 7$

$|\bar{a}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{4+9+1}$

$= \sqrt{14}$

$\Rightarrow \cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} = \dfrac{7}{\sqrt{14}\cdot\sqrt{14}} = \dfrac{7}{14} = \dfrac{1}{2}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right)$

* If $\bar{a} = (2,-3,-1)$ and $\bar{b} = (1,4,-3)$ then find $(\bar{a}+\bar{b}) \times (\bar{a}-\bar{b})$ also find modulus.

$\Rightarrow \bar{a}+\bar{b} = (2,-3,-1) + (1,4,-3)$

$= (2+1, -3+4, -1-3)$

$\bar{a}+\bar{b} = (3,1,-4)$

$\Rightarrow \bar{a}-\bar{b} = (2,-3,-1) - (1,4,-3)$

$= (2-1, -3-4, -1+3)$

$\bar{a}-\bar{b} = (1,-7,2)$

$(\bar{a}+\bar{b})\times(\bar{a}-\bar{b}) = \begin{vmatrix} i & j & k \\ 3 & 1 & -4 \\ 1 & -7 & 2 \end{vmatrix}$

$= i\begin{vmatrix} 1 & -4 \\ -7 & 2 \end{vmatrix} - j\begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} + k\begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix}$

$= i(2-28) - j(6+4) + k(-21-1)$

$= -26i - 10j - 22k$

$= (-26, -10, -22)$

Now, $|(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{(-26)^2+(-10)^2+(-22)^2}$

$= \sqrt{676+100+484}$

$= \sqrt{1260}$

*simplify:

$(10i+2j+3k)\cdot\left[(i-2j+2k)\times(3i-2j-2k)\right]$

Let

$\Rightarrow \bar{a} = (10,2,3)$, $\bar{b} = (1,-2,2)$, $\bar{c} = (3,-2,-2)$

$\bar{b}\times\bar{c} = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$

$= i\begin{vmatrix} -2 & 2 \\ -2 & -2 \end{vmatrix} - j\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} + k\begin{vmatrix} 1 & -2 \\ 3 & -2 \end{vmatrix}$

$= i(4+4) - j(-2-6) + k(-2+6)$

$= 8i + 8j + 4k$

$= (8,8,4)$

Now, $\bar{a}\cdot(\bar{b}\times\bar{c})$

$= (10,2,3)\cdot(8,8,4)$

$= 80+16+12$

$= 108$

* If $\bar{x} = (1,1,1)$ and $\bar{y} = (2,-1,-1)$ then prove that $\bar{x}$ is perpendicular to $\bar{y}$. Also find perpendicular unit vector to both $\bar{x}$ and $\bar{y}$.

$\Rightarrow \bar{x} = (1,1,1)$, $\bar{y} = (2,-1,-1)$

Now, $\bar{x}\cdot\bar{y} = (1,1,1)\cdot(2,-1,-1)$

$= 2-1-1$

$= 0$

$\bar{x}\cdot\bar{y} = 0 \Rightarrow \bar{x}$ is perpendicular to $\bar{y}$.

$\bar{x}\times\bar{y} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix}$

$= i\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$

$= i(-1+1) - j(-1-2) + k(-1-2)$

$= 0i + 3j - 3k$

$= (0,3,-3)$

$|\bar{x}\times\bar{y}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{0+9+9} = \sqrt{18}$.

P.U.V. to $\bar{x}$ & $\bar{y}$ $= \dfrac{\bar{x}\times\bar{y}}{|\bar{x}\times\bar{y}|}$

$= \dfrac{1}{\sqrt{18}}(0,3,-3)$.

* If $\bar{a} = i+j+k$ and $\bar{b} = 2i-2j+k$ then find perpendicular unit vector to both $\bar{a}$ and $\bar{b}$.

$\Rightarrow \bar{a} = (1,1,1)$, $\bar{b} = (2,-2,1)$

$\Rightarrow \bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$

$= i\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix}$

$= i(1+2) - j(1-2) + k(-2-2)$

$= 3i + j - 4k$

$= (3, 1, -4)$

$\Rightarrow |\bar{a}\times\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{9+1+16}$

$= \sqrt{26}$

P.U.V. to $\bar{a}$ & $\bar{b}$ $= \dfrac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}$

$= \dfrac{1}{\sqrt{26}}(3, 1, -4)$

* If $\bar{a} = 2i - 3j + 4k$ and $\bar{b} = i - j + k$ then find perpendicular unit vector to both $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$.

$\Rightarrow \bar{a} = (2, -3, 4)$ $\quad \bar{b} = (1, -1, 1)$

$\Rightarrow \bar{a}+\bar{b} = (2, -3, 4) + (1, -1, 1)$

$= (2+1, -3-1, 4+1)$

$= (3, -4, 5)$

$\Rightarrow \bar{a}-\bar{b} = (2, -3, 4) - (1, -1, 1)$

$= (2-1, -3+1, 4-1)$

$= (1, -2, 3)$

$\Rightarrow (\bar{a}+\bar{b})\times(\bar{a}-\bar{b}) = \begin{vmatrix} i & j & k \\ 3 & -4 & 5 \\ 1 & -2 & 3 \end{vmatrix}$

$= i\begin{vmatrix} -4 & 5 \\ -2 & 3 \end{vmatrix} - j\begin{vmatrix} 3 & 5 \\ 1 & 3 \end{vmatrix} + k\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix}$

$= i(-12+10) - j(9-5) + k(-6+4)$

$= -2i - 4j - 2k$

$= (-2, -4, -2)$

$|(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{4+16+4} = \sqrt{24}$

P.U.V. to $(\bar{a}+\bar{b})$ and $(\bar{a}-\bar{b})$ $= \dfrac{(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})}{|(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})|}$

$= \dfrac{1}{\sqrt{24}}(-2, -4, -2)$

* A Particle moves from the point $3i - 2j + k$ to the point $i + 3j - 4k$ under the effect of constant forces $i - j + k$, $i + j - 3k$ and $4i + 5j - 6k$. Find the work done.

$\Rightarrow F_1 = (1, -1, 1)$ $\qquad d_1 = (3, -2, 1)$

$F_2 = (1, 1, -3)$ $\qquad d_2 = (1, 3, -4)$

$F_3 = (4, 5, -6)$

$F = F_1 + F_2 + F_3$

$= (1, -1, 1) + (1, 1, -3) + (4, 5, -6)$

$= (1+1+4, -1+1+5, 1-3-6)$

$= (6, 5, -8)$

$d = d_2 - d_1$

$= (1, 3, -4) - (3, -2, 1)$

$= (1-3, 3+2, -4-1)$

$= (-2, 5, -5)$

$W = F \cdot d$

$= (6, 5, -8) \cdot (-2, 5, -5)$

$= -12 + 25 + 40$

$= 53$ unit.

* Forces $3i - j + 2k$ and $i + 3j - k$ act on a particle and particle moves from the point $2i + 3j + k$ to the point $5i + 2j + 3k$ under the effect of these forces. Find the work done.

$\Rightarrow F_1 = (3, -1, 2)$ $\qquad d_1 = (2, 3, 1)$

$F_2 = (1, 3, -1)$ $\qquad d_2 = (5, 2, 3)$

$\Rightarrow F = F_1 + F_2$

$= (3, -1, 2) + (1, 3, -1)$

$= (3+1, -1+3, 2-1)$

$F = (4,2,1)$

$\Rightarrow d = d_2 - d_1$

$= (5,2,3) - (2,3,1)$

$= (5-2, 2-3, 3-1)$

$= (3,-1,2)$

$\Rightarrow W = F \cdot d$

$= (4,2,1) \cdot (3,-1,2)$

$= 12-2+2$

$= 12$ unit.

* Prove that $\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2$

$$L.H.S = \frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24}$$

$$= \frac{1}{\frac{\log 24}{\log 6}} + \frac{1}{\frac{\log 24}{\log 12}} + \frac{1}{\frac{\log 24}{\log 8}}$$

$$= \frac{\log 6}{\log 24} + \frac{\log 12}{\log 24} + \frac{\log 8}{\log 24}$$

$$= \frac{\log 6 + \log 12 + \log 8}{\log 24}$$

$$= \frac{\log (6 \times 12 \times 8)}{\log 24}$$

$$= \frac{\log 576}{\log 24}$$

$$= \frac{\log 24^2}{\log 24}$$

$$= \frac{2 \log 24}{\log 24}$$

$= 2 = R.H.S.$

* Prove that :-

$$\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} = 2$$

$$L.H.S = \frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz}$$

$$= \frac{1}{\frac{\log xyz}{\log xy}} + \frac{1}{\frac{\log xyz}{\log yz}} + \frac{1}{\frac{\log xyz}{\log zx}}$$

$$= \frac{\log xy}{\log xyz} + \frac{\log yz}{\log xyz} + \frac{\log zx}{\log xyz}$$

$$= \frac{\log xy + \log yz + \log zx}{\log xyz}$$

$$= \frac{\log (xy \cdot yz \cdot zx)}{\log xyz}$$

$$= \frac{\log (xyz)^2}{\log xyz}$$

$$= \frac{2 \log xyz}{\log xyz}$$

$= 2 = R.H.S.$

* Prove that :-

$$\frac{1}{\log_x yz + 1} + \frac{1}{\log_y zx + 1} + \frac{1}{\log_z xy + 1} = 1$$

$$L.H.S = \frac{1}{\log_x yz + 1} + \frac{1}{\log_y zx + 1} + \frac{1}{\log_z xy + 1}$$

$$= \frac{1}{\frac{\log yz}{\log x} + 1} + \frac{1}{\frac{\log zx}{\log y} + 1} + \frac{1}{\frac{\log xy}{\log z} + 1}$$

$$= \frac{1}{\frac{\log yz + \log x}{\log x}} + \frac{1}{\frac{\log zx + \log y}{\log y}} + \frac{1}{\frac{\log xy + \log z}{\log z}}$$

$$= \frac{\log x}{\log xyz} + \frac{\log y}{\log xyz} + \frac{\log z}{\log xyz}$$

$$= \frac{\log x + \log y + \log z}{\log xyz}$$

$$= \frac{\log(xyz)}{\log xyz}$$

$$= 1 = R.H.S.$$

* Prove that $\log\left[x+\sqrt{x^2-1}\right] + \log\left[x-\sqrt{x^2-1}\right] = 0$

$L.H.S = \log\left[x+\sqrt{x^2-1}\right] + \log\left[x-\sqrt{x^2-1}\right]$

$= \log\left[\left(x+\sqrt{x^2-1}\right)\cdot\left(x-\sqrt{x^2-1}\right)\right]$

$= \log\left[x^2 - \left(\sqrt{x^2-1}\right)^2\right]$

$= \log\left[x^2-(x^2-1)\right]$

$= \log\left[x^2-x^2+1\right]$

$= \log 1 = 0 = R.H.S.$

* If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then prove that $a=b$.

$\Rightarrow \log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\Rightarrow 2\log\left(\frac{a+b}{2}\right) = \log(ab)$

$\Rightarrow \log\left(\frac{a+b}{2}\right)^2 = \log(ab)$

$\Rightarrow \left(\frac{a+b}{2}\right)^2 = ab$

$\Rightarrow \frac{a^2+2ab+b^2}{4} = ab$

$\Rightarrow a^2+2ab+b^2 = 4ab$

$\Rightarrow a^2+b^2 = 4ab-2ab$

$\Rightarrow a^2+b^2 = 2ab$

$\Rightarrow a^2-2ab+b^2 = 0$

$\Rightarrow (a-b)^2 = 0$

$\Rightarrow a-b = 0$

$\Rightarrow a = b.$

* If $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then Prove that $a^2+b^2 = 6ab$ or $\frac{a}{b}+\frac{b}{a} = 6$

$\Rightarrow \log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\Rightarrow 2\log\left(\frac{a-b}{2}\right) = \log(ab)$

$\Rightarrow \log\left(\frac{a-b}{2}\right)^2 = \log(ab)$

$\Rightarrow \left(\frac{a-b}{2}\right)^2 = ab$

$\Rightarrow \frac{a^2-2ab+b^2}{4} = ab$

$\Rightarrow a^2-2ab+b^2 = 4ab$

$\Rightarrow a^2+b^2 = 4ab+2ab$

$\Rightarrow a^2+b^2 = 6ab$

$\Rightarrow \frac{a^2}{ab}+\frac{b^2}{ab} = \frac{6ab}{ab}$

$\Rightarrow \frac{a}{b}+\frac{b}{a} = 6.$

* If $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$ then Prove that $x^2+y^2 = 7xy$ or $\frac{x}{y}+\frac{y}{x} = 7$

$\Rightarrow \log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$

$\Rightarrow 2\log\left(\frac{x+y}{3}\right) = \log(xy)$

$\Rightarrow \log\left(\frac{x+y}{3}\right)^2 = \log(xy)$

$\Rightarrow \left(\frac{x+y}{3}\right)^2 = xy$

$\Rightarrow \frac{x^2+2xy+y^2}{9} = xy$

$\Rightarrow x^2+2xy+y^2 = 9xy$

$\Rightarrow x^2+y^2 = 9xy-2xy$

$\Rightarrow x^2+y^2 = 7xy.$

$\Rightarrow \frac{x^2}{xy}+\frac{y^2}{xy} = \frac{7xy}{xy}$

$\Rightarrow \frac{x}{y}+\frac{y}{x} = 7.$